6.864: Lecture 20 (November 22, 2005)

Global Linear Models

Overview

- A brief review of history-based methods
- A new framework: Global linear models
- Parsing problems in this framework:
  Reranking problems
- Parameter estimation method 1:
  A variant of the perceptron algorithm

Techniques

- So far:
  - Smoothed estimation
  - Probabilistic context-free grammars
  - The EM algorithm
  - Log-linear models
  - Hidden markov models
  - History-based models
  - Partially supervised methods
- Today:
  - Global linear models

Supervised Learning in Natural Language

- General task: induce a function F from members of a set $\mathcal{X}$ to members of a set $\mathcal{Y}$. e.g.,

Problem	$x \in \mathcal{X}$	$y \in \mathcal{Y}$
Parsing	sentence	parse tree
Machine translation	French sentence	English sentence
POS tagging	sentence	sequence of tags

- Supervised learning:
  we have a *training set* (x_i, y_i) for $i = 1 \ldots n$

The Models so far

- Most of the models we've seen so far are **history-based models**:
  - We break structures down into a derivation, or sequence of decisions
  - Each decision has an associated conditional probability
  - Probability of a structure is a product of decision probabilities
  - Parameter values are estimated using variants of maximum-likelihood estimation
  - Function $F : \mathcal{X} \rightarrow \mathcal{Y}$ is defined as

$$F(x) = \text{argmax}_y P(y, x \mid \Theta) \quad \text{or} \quad F(x) = \text{argmax}_y P(y \mid x, \Theta)$$

Example 1: PCFGs

- We break structures down into a derivation, or sequence of decisions
  We have a top-down derivation, where each decision is to expand some non-terminal α with a rule $\alpha \rightarrow \beta$

- Each decision has an associated conditional probability
  $\alpha \rightarrow \beta$ has probability $P(\alpha \rightarrow \beta \mid \alpha)$

- Probability of a structure is a product of decision probabilities

$$P(T, S) = \prod_{i=1}^{n} P(\alpha_i \rightarrow \beta_i \mid \alpha_i)$$

  where $\alpha_i \rightarrow \beta_i$ for $i = 1 \ldots n$ are the n rules in the tree

- Parameter values are estimated using variants of maximum-likelihood estimation

$$P(\alpha \rightarrow \beta \mid \alpha) = \frac{Count(\alpha \rightarrow \beta)}{Count(\alpha)}$$

- Function $F : \mathcal{X} \rightarrow \mathcal{Y}$ is defined as

$$F(x) = \text{argmax}_y P(y, x \mid \Theta)$$

Can be computed using dynamic programming

Example 2: Log-linear Taggers

- We break structures down into a derivation, or sequence of decisions
  For a sentence of length n we have n tagging decisions, in left-to-right order

- Each decision has an associated conditional probability

$$P(t_i \mid t_{i-1}, t_{i-2}, w_1 \ldots w_n)$$

  where t_i is the i'th tagging decision, w_i is the i'th word

- Probability of a structure is a product of decision probabilities

$$P(t_1 \ldots t_n \mid w_1 \ldots w_n) = \prod_{i=1}^{n} P(t_i \mid t_{i-1}, t_{i-2}, w_1 \ldots w_n)$$

- Parameter values are estimated using variants of maximum-likelihood estimation
  $P(t_i \mid t_{i-1}, t_{i-2}, w_1 \ldots w_n)$ is estimated using a log-linear model

- Function $F : \mathcal{X} \rightarrow \mathcal{Y}$ is defined as

$$F(x) = \text{argmax}_y P(y \mid x, \Theta)$$

Can be computed using dynamic programming

Example 3: Machine Translation

- We break structures down into a derivation, or sequence of decisions
  A French sentence $\mathbf{f}$ is generated from an English sentence $\mathbf{e}$ in a number of steps: pick alignment for each French word, pick the French word given the English word

- Each decision has an associated conditional probability
  e.g., $\mathbf{T}(le \mid the)$, $\mathbf{D}(4 \mid 3, 6, 7)$

- Probability of a structure is a product of decision probabilities
  $P(\mathbf{f}, \mathbf{a} \mid \mathbf{e})$ is a product of translation and alignment probabilities

- Parameter values are estimated using variants of maximum-likelihood estimation
  Some decisions are **hidden**, so we use EM

- Function $F : \mathcal{X} \rightarrow \mathcal{Y}$ is defined as

$$F(\mathbf{f}) = \text{argmax}_{\mathbf{e},\mathbf{a}} P(\mathbf{e}) P(\mathbf{f}, \mathbf{a} \mid \mathbf{e})$$

  Approximated using greedy search methods

A New Set of Techniques: Global Linear Models

Overview of today's lecture:

- Global linear models as a framework
- Parsing problems in this framework:
  - Reranking problems
- A variant of the perceptron algorithm

Global Linear Models as a Framework

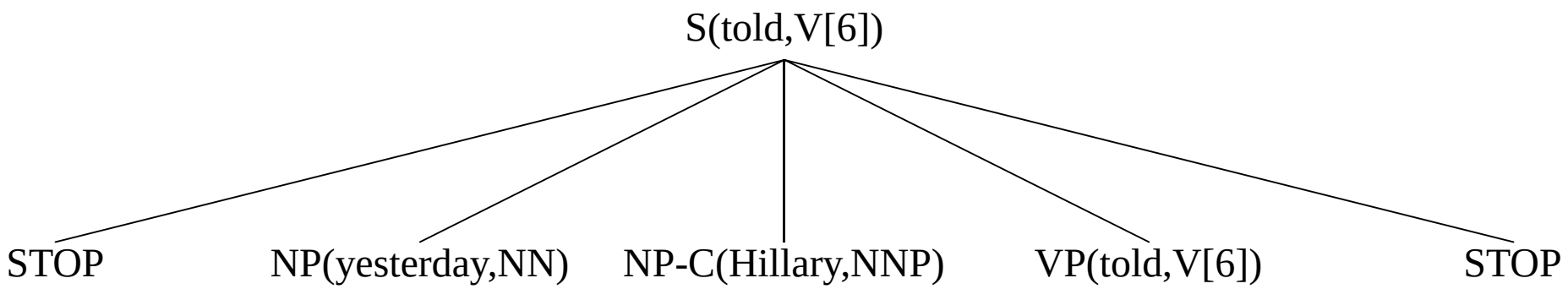
- We'll move away from history-based models
  No idea of a "derivation", or attaching probabilities to "decisions"

- Instead, we'll have feature vectors over entire structures
  "Global features"

- First piece of motivation:
  Freedom in defining features

An Example: Parsing

- In lecture 4, we described lexicalized models for parsing
- Showed how a rule can be generated in a number of steps



Model 2

- Step 1: generate category of head child

S(told,V[6])

$\Downarrow$

S(told,V[6])

|

VP(told,V[6])

$P_h(\text{VP} \mid \text{S, told, V[6]})$

Model 2

- Step 2: choose left **subcategorization frame**

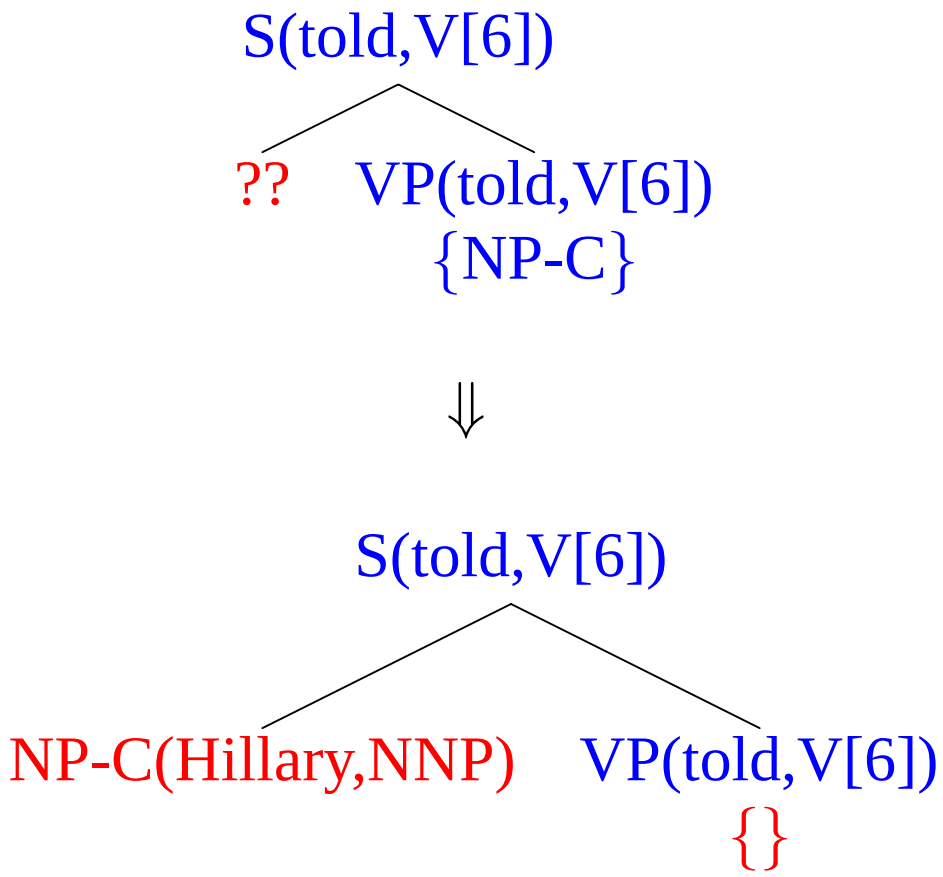
S(told,V[6])
|
VP(told,V[6])

⇓

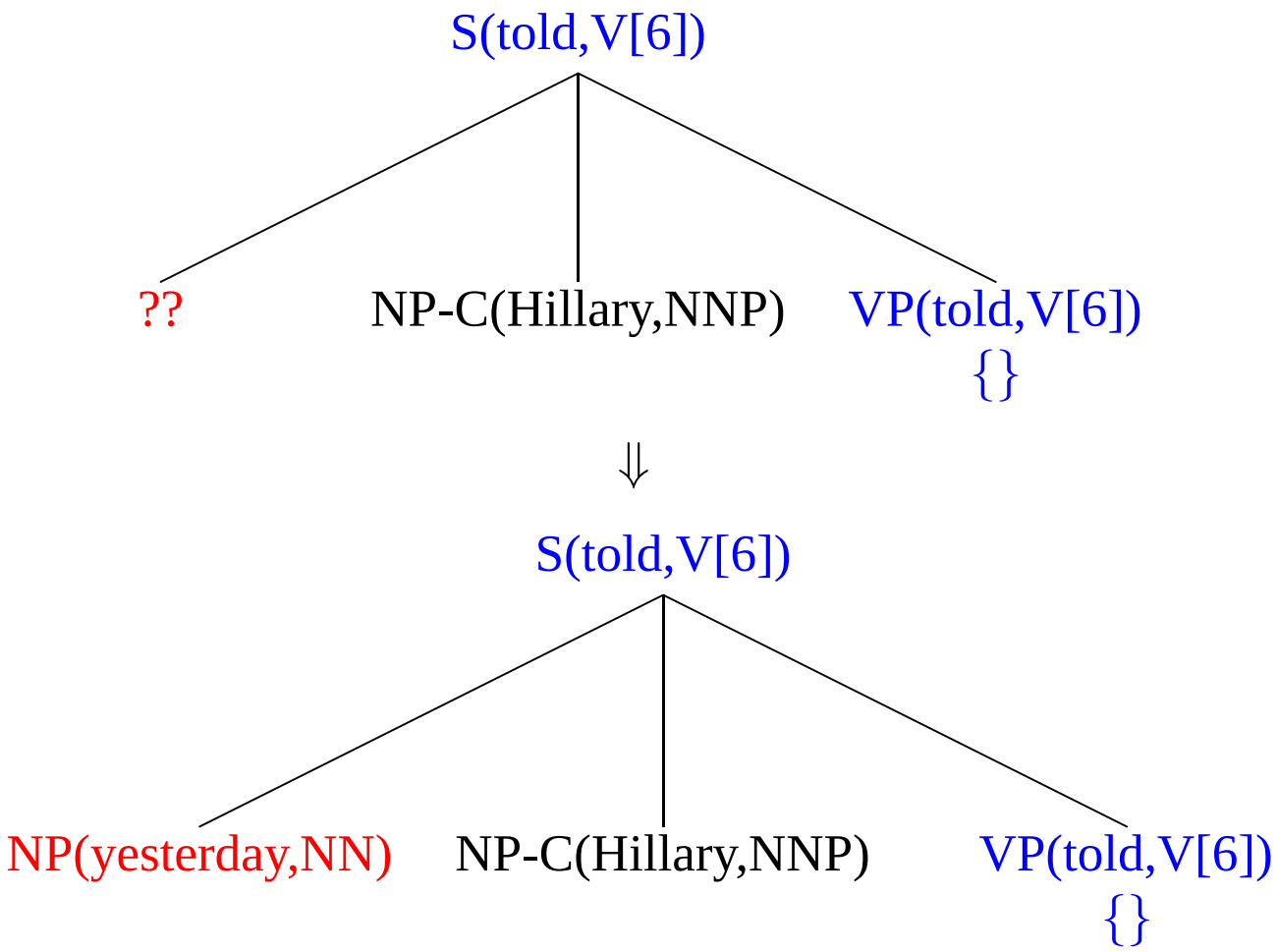
S(told,V[6])
|
VP(told,V[6])
{NP-C}

$P_h(\text{VP} \mid \text{S, told, V[6]}) \times P_{lc}(\{\text{NP-C}\} \mid \text{S, VP, told, V[6]})$

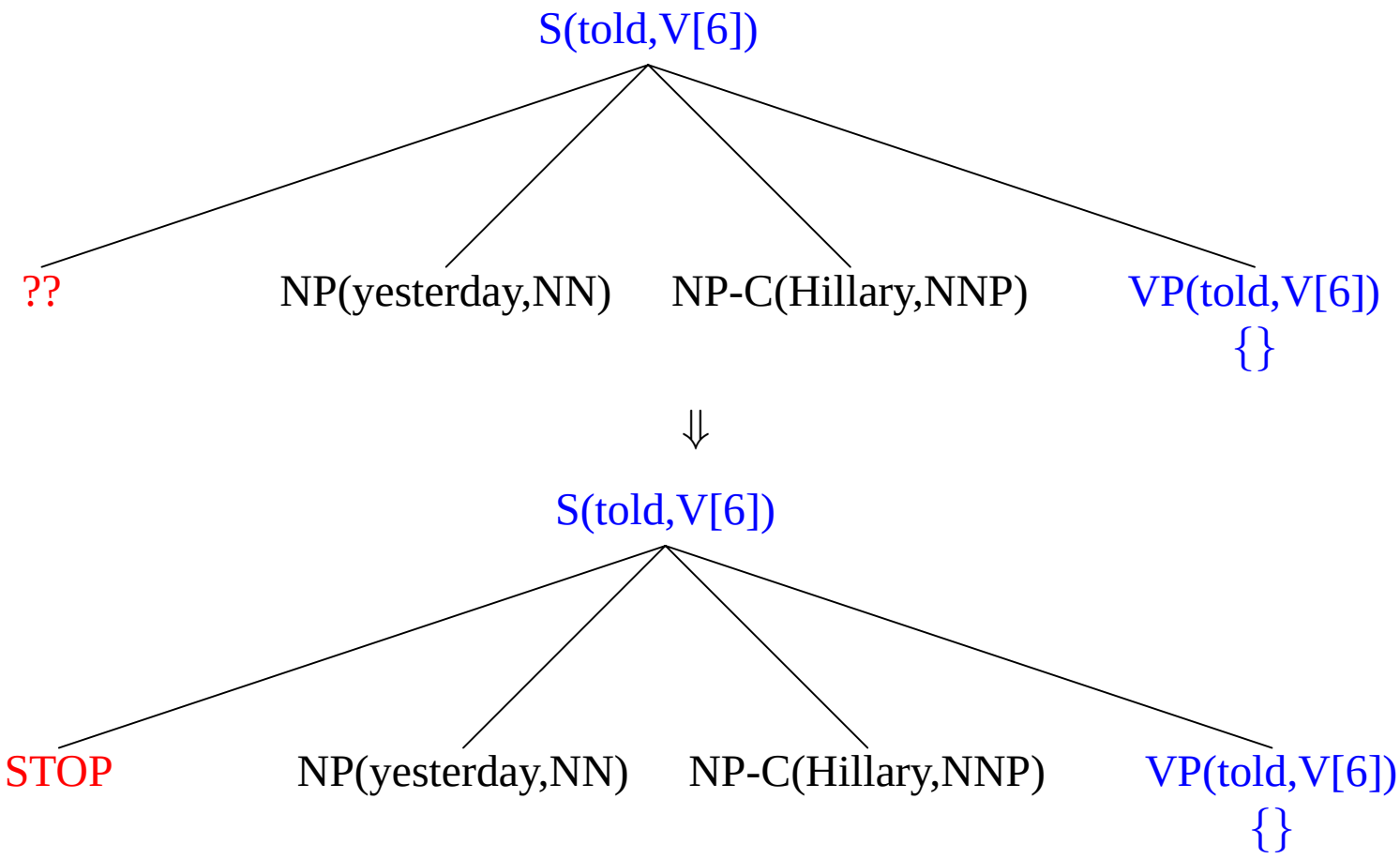
- Step 3: generate left modifiers in a Markov chain



$P_h(\text{VP} \mid \text{S, told, V[6]}) \times P_{lc}(\{\text{NP-C}\} \mid \text{S, VP, told, V[6]}) \times$
$P_d(\text{NP-C(Hillary,NNP)} \mid \text{S,VP,told,V[6],LEFT,}\{\text{NP-C}\})$

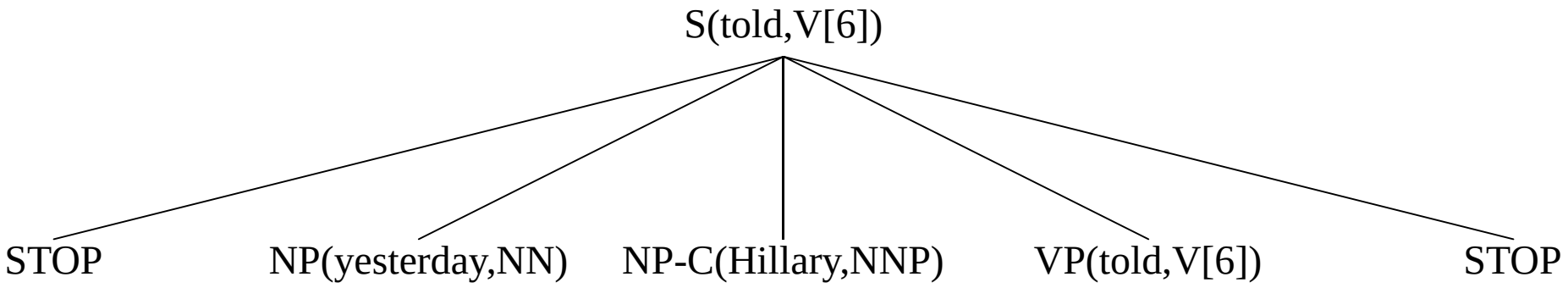


$P_h(\text{VP} \mid \text{S, told, V[6]}) \times P_{lc}(\{\text{NP-C}\} \mid \text{S, VP, told, V[6]})$
$P_d(\text{NP-C(Hillary,NNP)} \mid \text{S,VP,told,V[6],LEFT,}\{\text{NP-C}\}) \times$
$P_d(\text{NP(yesterday,NN)} \mid \text{S,VP,told,V[6],LEFT,}\{\})$



$P_h(\text{VP} \mid \text{S, told, V[6]}) \times P_{lc}(\{\text{NP-C}\} \mid \text{S, VP, told, V[6]})$
$P_d(\text{NP-C(Hillary,NNP)} \mid \text{S,VP,told,V[6],LEFT,}\{\text{NP-C}\}) \times$
$P_d(\text{NP(yesterday,NN)} \mid \text{S,VP,told,V[6],LEFT,}\{\}) \times$
$P_d(\text{STOP} \mid \text{S,VP,told,V[6],LEFT,}\{\})$

The Probabilities for One Rule



$P_h(\text{VP} \mid \text{S, told, V[6]}) \times$
$P_{lc}(\{\text{NP-C}\} \mid \text{S, VP, told, V[6]}) \times$
$P_{rc}(\{\} \mid \text{S, VP, told, V[6]}) \times$
$P_d(\text{NP-C(Hillary,NNP)} \mid \text{S,VP,told,V[6],LEFT,}\Delta = 1,\{\text{NP-C}\}) \times$
$P_d(\text{NP(yesterday,NN)} \mid \text{S,VP,told,V[6],LEFT,}\Delta = 0,\{\}) \times$
$P_d(\text{STOP} \mid \text{S,VP,told,V[6],LEFT,}\Delta = 0,\{\}) \times$
$P_d(\text{STOP} \mid \text{S,VP,told,V[6],RIGHT,}\Delta = 1,\{\})$

Three parameter types:
Head parameters, Subcategorization parameters, Dependency parameters

Smoothed Estimation

$$P(\text{NP(\_\_,NN) VP} \mid \text{S(questioned,Vt)}) =$$

$$\lambda_1 \times \frac{Count(\text{S(questioned,Vt)} \rightarrow \text{NP(\_\_,NN) VP})}{Count(\text{S(questioned,Vt)})}$$

$$+\lambda_2 \times \frac{Count(\text{S(\_\_,Vt)} \rightarrow \text{NP(\_\_,NN) VP})}{Count(\text{S(\_\_,Vt)})}$$

- Where $0 \leq \lambda_1, \lambda_2 \leq 1$, and $\lambda_1 + \lambda_2 = 1$

Smoothed Estimation

$$P(\text{lawyer} \mid \text{S,VP,NP,NN,questioned,Vt}) =$$

$$\lambda_1 \times \frac{Count(\text{lawyer} \mid \text{S,VP,NP,NN,questioned,Vt})}{Count(\text{S,VP,NP,NN,questioned,Vt})}$$

$$+\lambda_2 \times \frac{Count(\text{lawyer} \mid \text{S,VP,NP,NN,Vt})}{Count(\text{S,VP,NP,NN,Vt})}$$

$$+\lambda_3 \times \frac{Count(\text{lawyer} \mid \text{NN})}{Count(\text{NN})}$$

- Where $0 \leq \lambda_1, \lambda_2, \lambda_3 \leq 1$, and $\lambda_1 + \lambda_2 + \lambda_3 = 1$

An Example: Parsing

- In lecture 4, we described lexicalized models for parsing
- Showed how a rule can be generated in a number of steps
- The end result:
  - We have head, dependency, and subcategorization parameters
  - Smoothed estimation means "probability" of a tree depends on counts of many different types of events
- **What if we want to add new features?**
  Can be very awkward to incorporate some features in history-based models

A Need for Flexible Features

Example 1 Parallelism in coordination [Johnson et. al 1999]

Constituents with similar structure tend to be coordinated
$\Rightarrow$ how do we allow the parser to learn this preference?

Bars in New York and pubs in London
vs. Bars in New York and pubs

Example 2 Semantic features

We might have an ontology giving properties of various nouns/verbs

$\Rightarrow$ how do we allow the parser to use this information?

pour the **cappucino**

vs. pour the **book**

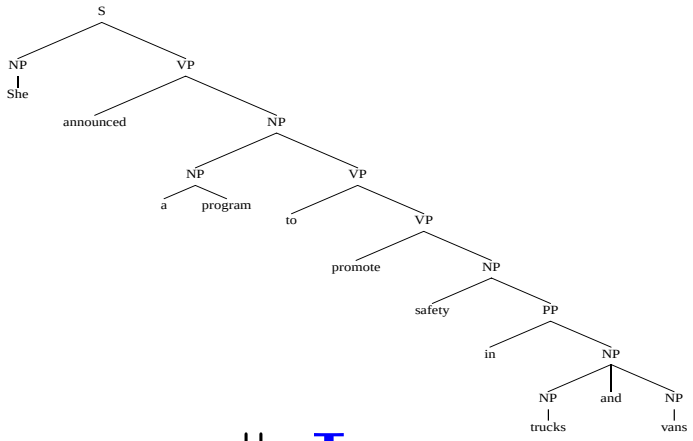
Ontology states that **cappucino** has the `+liquid` feature, **book** does not.

Three Components of Global Linear Models

- $\mathbf{\Phi}$ is a function that maps a structure (x, y) to a **feature vector** $\mathbf{\Phi}(x, y) \in \mathbb{R}^d$
- $\mathbf{GEN}$ is a function that maps an input x to a set of **candidates** $\mathbf{GEN}(x)$
- $\mathbf{W}$ is a parameter vector (also a member of $\mathbb{R}^d$)
- Training data is used to set the value of $\mathbf{W}$

Component 1: Φ

- Φ maps a candidate to a **feature vector** $\in \mathbb{R}^d$
- Φ defi nes the **representation** of a candidate

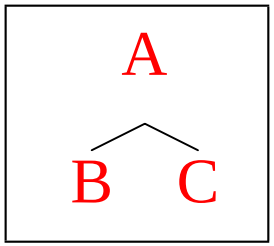


$\Downarrow \Phi$

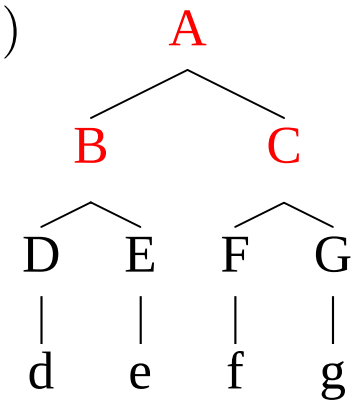
$\langle 1, 0, 2, 0, 0, 15, 5 \rangle$

Features

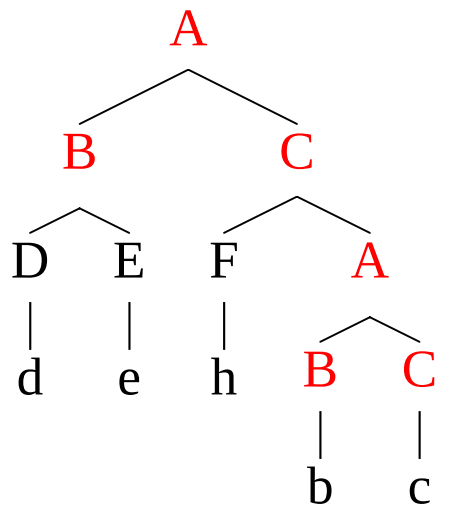
- A "feature" is a function on a structure, e.g.,

$h(x, y) =$ Number of times  is seen in (x, y)

(x_1, y_1)



(x_2, y_2)



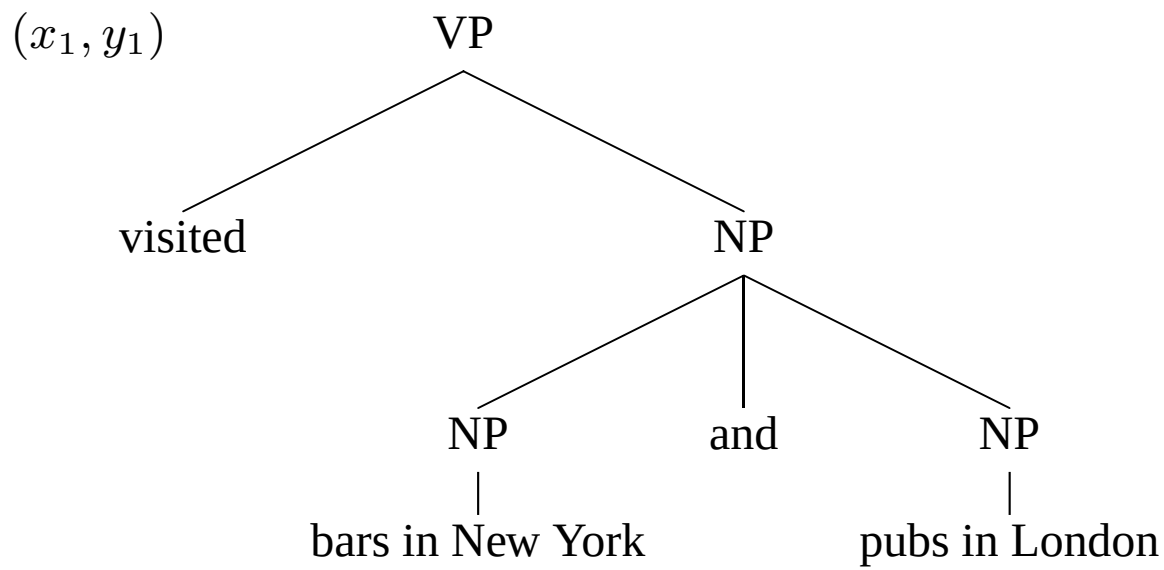
$h(x_1, y_1) = 1$

$h(x_2, y_2) = 2$

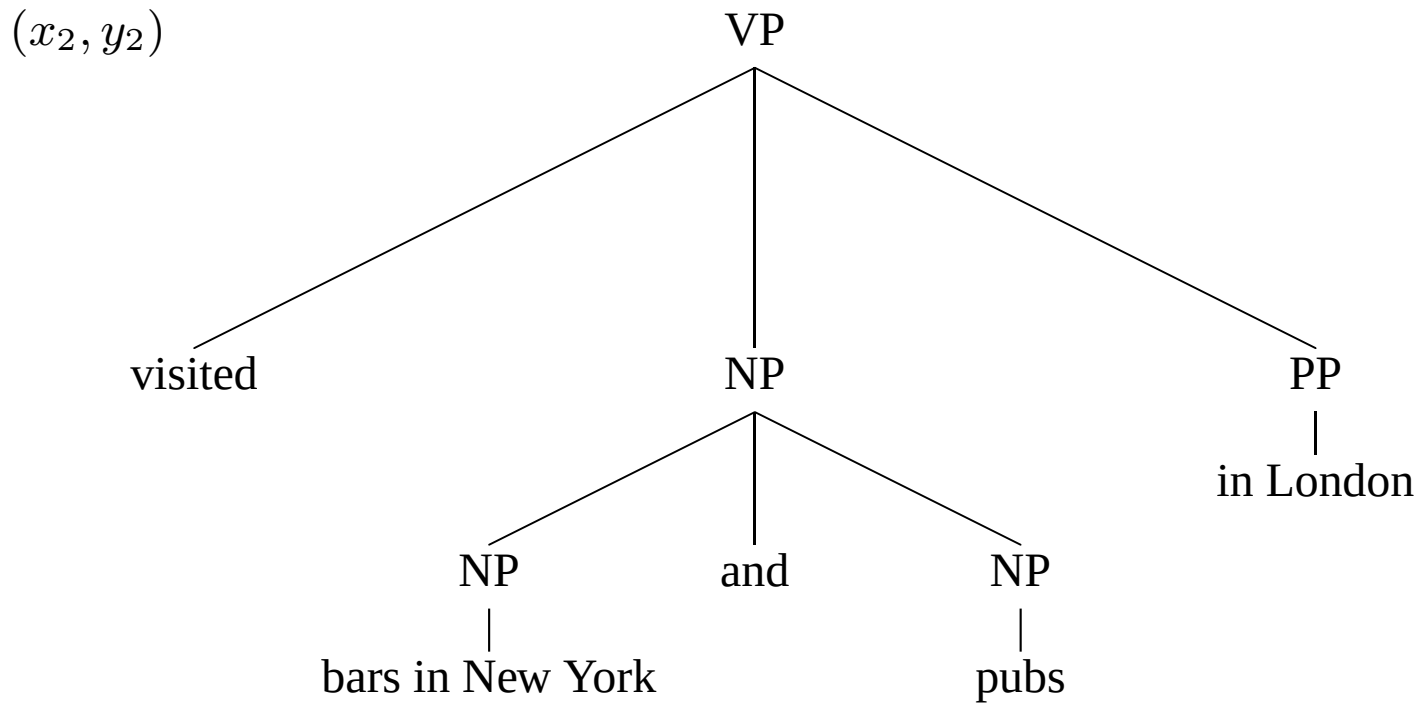
Another Example

- A "feature" is a function on a structure, e.g.,

$$h(x, y) = \begin{cases} 1 & \text{if } (x, y) \text{ has an instance of non-parallel coordination} \\ 0 & \text{otherwise} \end{cases}$$



$$h(x_1, y_1) = 0$$



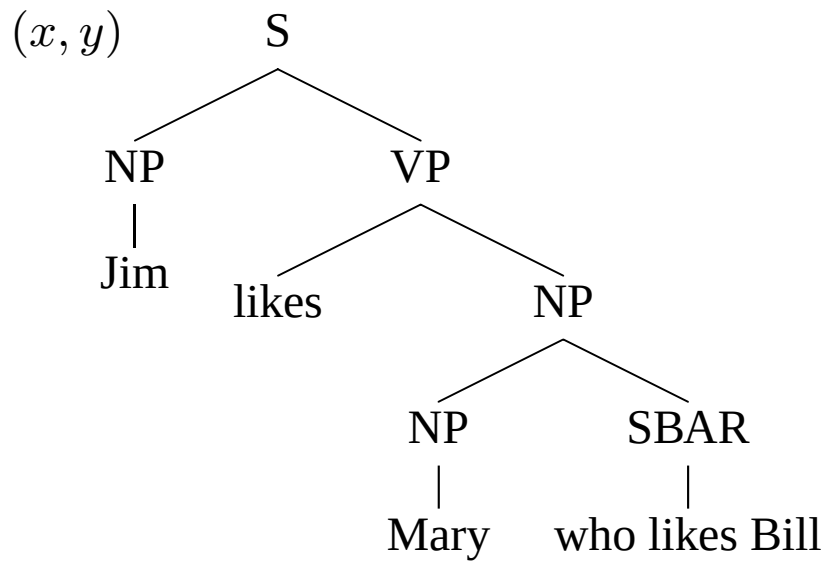
$$h(x_2, y_2) = 1$$

A Third Example

- A "feature" is a function on a structure, e.g.,

$h_1(x, y)$ = number of times *Mary* is subject of *likes*

$h_2(x, y)$ = number of times *Mary* is object of *likes*



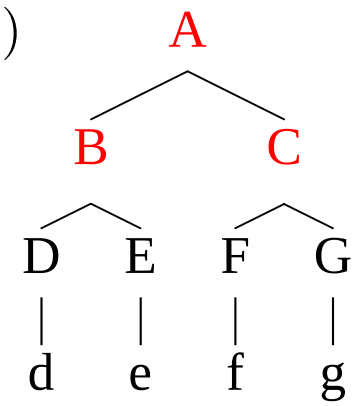
$h_1(x, y) = 1$ $\qquad$ $h_2(x, y) = 1$

A Final Example

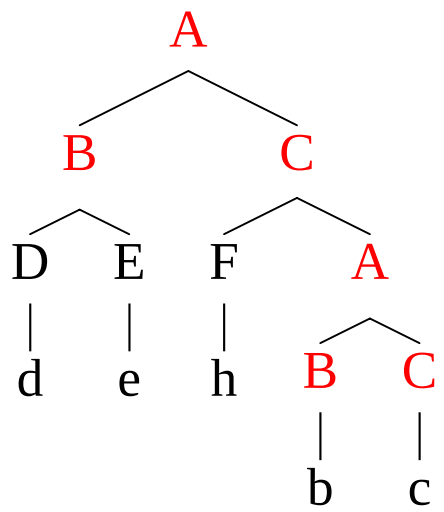
- A "feature" is a function on a structure, e.g.,

$$h(x, y) = \text{log probability of } (x, y) \text{ under Model 2}$$

(x_1, y_1)



(x_2, y_2)

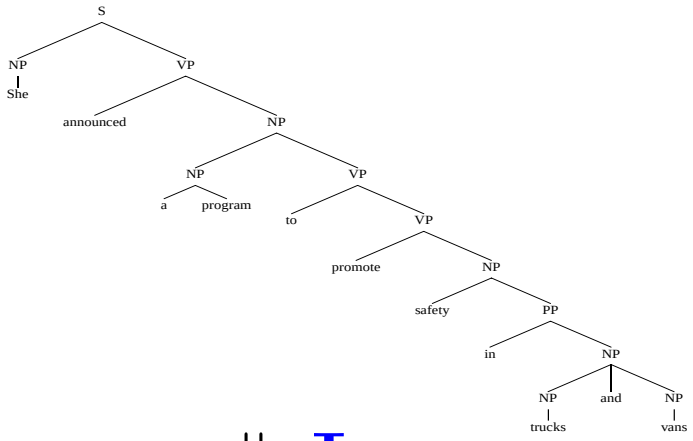


$$h(x_1, y_1) = -1.56$$

$$h(x_2, y_2) = -1.98$$

Component 1: Φ

- Φ maps a candidate to a **feature vector** $\in \mathbb{R}^d$
- Φ defi nes the **representation** of a candidate



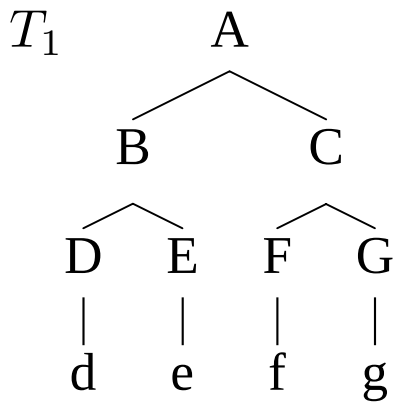
$\Downarrow \Phi$

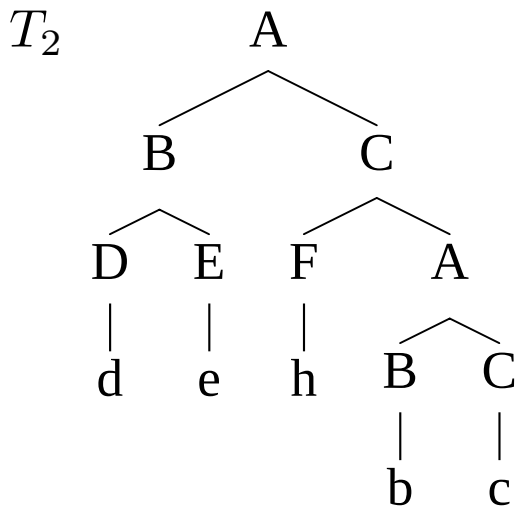
$\langle 1, 0, 2, 0, 0, 15, 5 \rangle$

Feature Vectors

- A set of functions $h_1 \ldots h_d$ defi ne a **feature vector**

$$\mathbf{\Phi}(x) = \langle h_1(x), h_2(x) \ldots h_d(x) \rangle$$





$$\mathbf{\Phi}(T_1) = \langle 1, 0, 0, 3 \rangle \qquad \mathbf{\Phi}(T_2) = \langle 2, 0, 1, 1 \rangle$$

Feature Vectors

- Our goal is to come up with learning methods which allow us to defi ne any features over parse trees

- Avoids the intermediate steps of a history-based model: defi ning a **derivation** which takes the features into account, and then attaching probabilities to **decisions**
  Our claim is that this can be an unwieldy, indirect way of getting desired features into a model

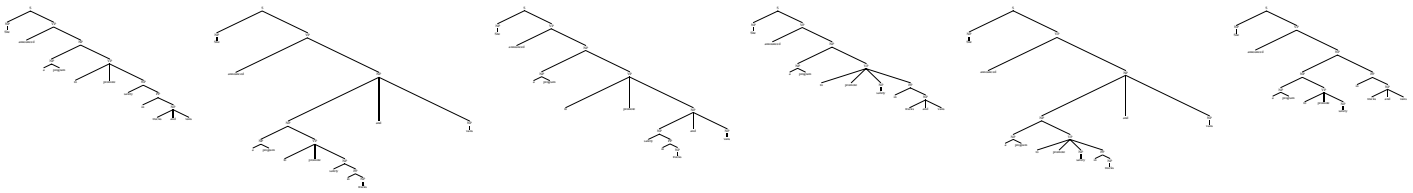
- Problem of **representation** now boils down to the choice of the function $\mathbf{\Phi}(x, y)$

Component 2: GEN

- **GEN** enumerates a set of **candidates** for a sentence

She announced a program to promote safety in trucks and vans

⇓ **GEN**



Component 2: GEN

- **GEN** enumerates a set of **candidates** for an input x

- Some examples of how $\mathbf{GEN}(x)$ can be defined:
  - Parsing: $\mathbf{GEN}(x)$ is the set of parses for x under a grammar
  - Any task: $\mathbf{GEN}(x)$ is the top N most probable parses under a history-based model
  - Tagging: $\mathbf{GEN}(x)$ is the set of all possible tag sequences with the same length as x
  - Translation: $\mathbf{GEN}(x)$ is the set of all possible English translations for the French sentence x

Component 3: $\mathbf{W}$

- $\mathbf{W}$ is a **parameter vector** $\in \mathbb{R}^d$
- $\mathbf{\Phi}$ and $\mathbf{W}$ together map a candidate to a real-valued score

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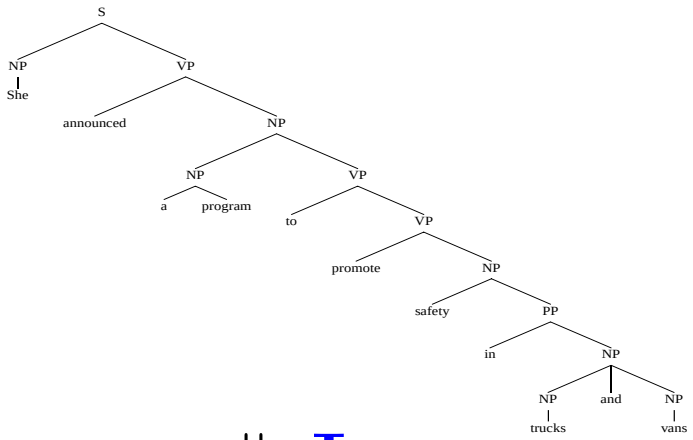


$$\Downarrow \mathbf{\Phi}$$

$$\langle 1, 0, 2, 0, 0, 15, 5 \rangle$$

$$\Downarrow \mathbf{\Phi} \cdot \mathbf{W}$$

$$\langle 1, 0, 2, 0, 0, 15, 5 \rangle \cdot \langle 1.9, -0.3, 0.2, 1.3, 0, 1.0, -2.3 \rangle = 5.8$$

Putting it all Together

- $\mathcal{X}$ is set of sentences, $\mathcal{Y}$ is set of possible outputs (e.g. trees)
- Need to learn a function $F : \mathcal{X} \rightarrow \mathcal{Y}$
- $\mathbf{GEN}$, $\mathbf{\Phi}$, $\mathbf{W}$ defi ne

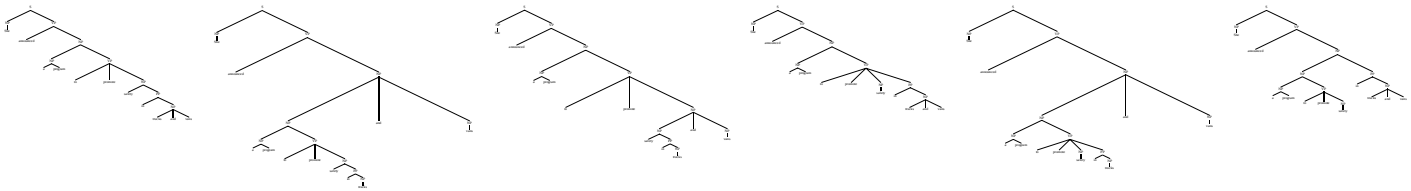
$$F(x) = \arg\max_{y \in \mathbf{GEN}(x)} \mathbf{\Phi}(x, y) \cdot \mathbf{W}$$

Choose the highest scoring candidate as the most plausible structure

- Given examples (x_i, y_i), how to set $\mathbf{W}$?

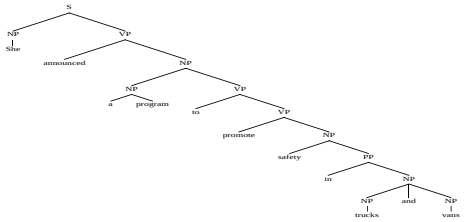
She announced a program to promote safety in trucks and vans

$\Downarrow$ **GEN**



$\Downarrow \Phi$	$\Downarrow \Phi$	$\Downarrow \Phi$	$\Downarrow \Phi$	$\Downarrow \Phi$	$\Downarrow \Phi$
$\langle 1, 1, 3, 5 \rangle$	$\langle 2, 0, 0, 5 \rangle$	$\langle 1, 0, 1, 5 \rangle$	$\langle 0, 0, 3, 0 \rangle$	$\langle 0, 1, 0, 5 \rangle$	$\langle 0, 0, 1, 5 \rangle$
$\Downarrow \Phi \cdot W$	$\Downarrow \Phi \cdot W$	$\Downarrow \Phi \cdot W$	$\Downarrow \Phi \cdot W$	$\Downarrow \Phi \cdot W$	$\Downarrow \Phi \cdot W$
13.6	12.2	12.1	3.3	9.4	11.1

$\Downarrow \arg\max$



Overview

- A brief review of history-based methods
- A new framework: Global linear models
- Parsing problems in this framework:
  Reranking problems
- Parameter estimation method 1:
  A variant of the perceptron algorithm

Reranking Approaches to Parsing

- Use a **baseline** parser to produce top N parses for each sentence in training and test data
  $\mathbf{GEN}(x)$ is the top N parses for x under the baseline model

- One method: use Model 2 to generate a number of parses
  (in our experiments, around 25 parses on average for 40,000 training sentences, giving $\approx$ 1 million training parses)

- **Supervision:** for each x_i take y_i to be the parse that is "closest" to the treebank parse in $\mathbf{GEN}(x_i)$

The Representation Φ

- Each component of Φ could be essentially *any* feature over parse trees

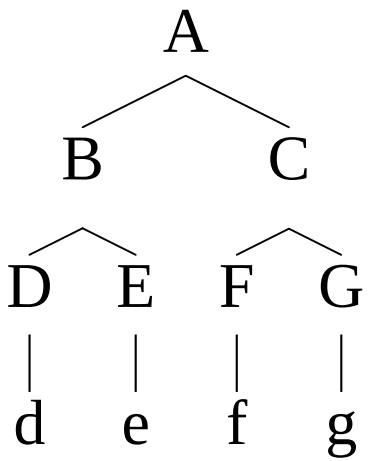
- For example:

  $\Phi_1(x, y) =$ log probability of (x, y) under the baseline model

  $$\Phi_2(x, y) = \begin{cases} 1 & \text{if } (x, y) \text{ includes the rule } \texttt{VP} \rightarrow \texttt{PP VBD NP} \\ 0 & \text{otherwise} \end{cases}$$

Practical Issues in Creating Φ

- Step 1: map a tree to a number of "strings" representing features

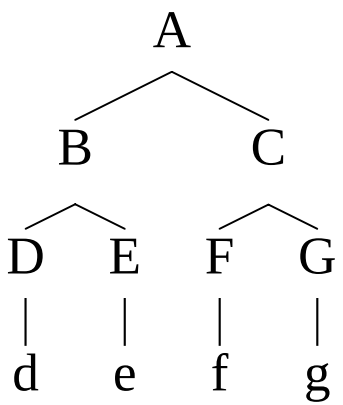


$\Rightarrow$

```
HASRULE:A->B;C
HASRULE:B->D;E
HASRULE:C->F;G
HASRULE:D->d
HASRULE:E->e
HASRULE:F->f
HASRULE:G->g
```

Practical Issues in Creating Φ

- Step 2: hash the strings to integers



$\Rightarrow$

HASRULE:A->B;C	54
HASRULE:B->D;E	118
HASRULE:C->F;G	14
HASRULE:D->d	10078
HASRULE:E->e	9000
HASRULE:F->f	1078
HASRULE:G->g	101

- In our experiments, tree is then represented as log probability under the baseline model, plus a sparse feature array:

$\mathbf{\Phi}_1(x, y) =$ log probability of (x, y) under the baseline model

$\mathbf{\Phi}_i(x, y) = 1$ for $i = \{54, 118, 14, 10078, 9000, 1078, 101\}$

$\mathbf{\Phi}_i(x, y) = 0$ for all other i

The Score for Each Parse

- In our experiments, tree is then represented as log probability under the baseline model, plus a sparse feature array:

$\mathbf{\Phi}_1(x, y) =$ log probability of (x, y) under the baseline model

$\mathbf{\Phi}_i(x, y) = 1$ for $i = \{54, 118, 14, 10078, 9000, 1078, 101\}$

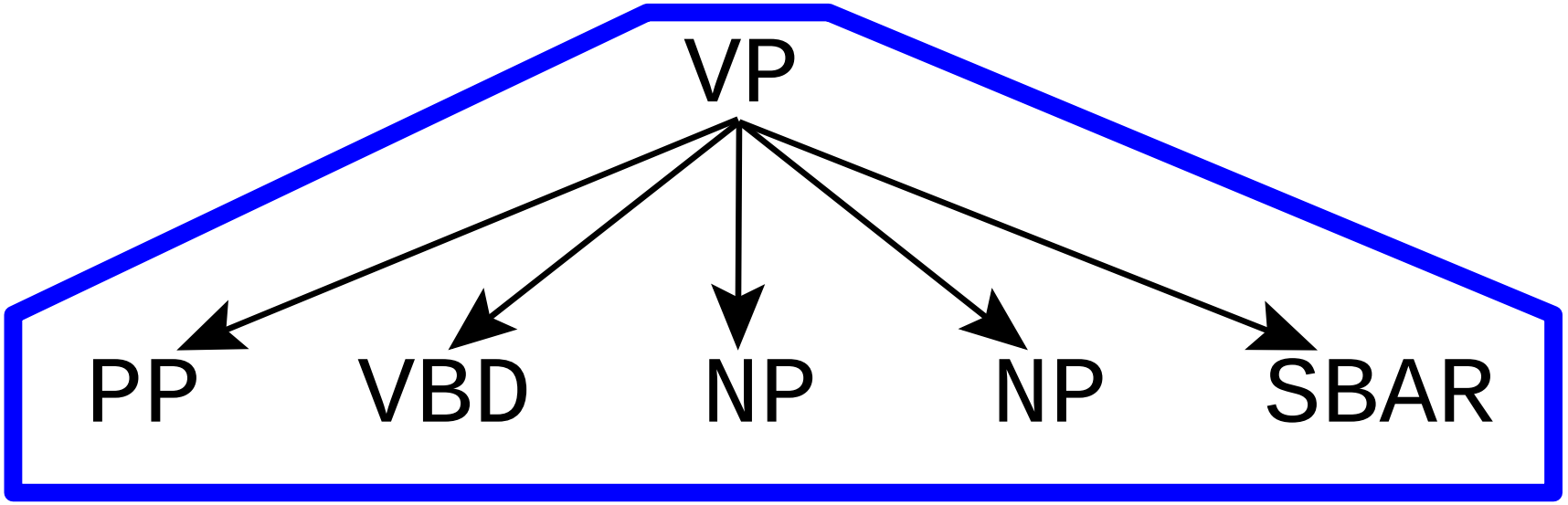
$\mathbf{\Phi}_i(x, y) = 0$ for all other i

- Score for the tree (x, y):

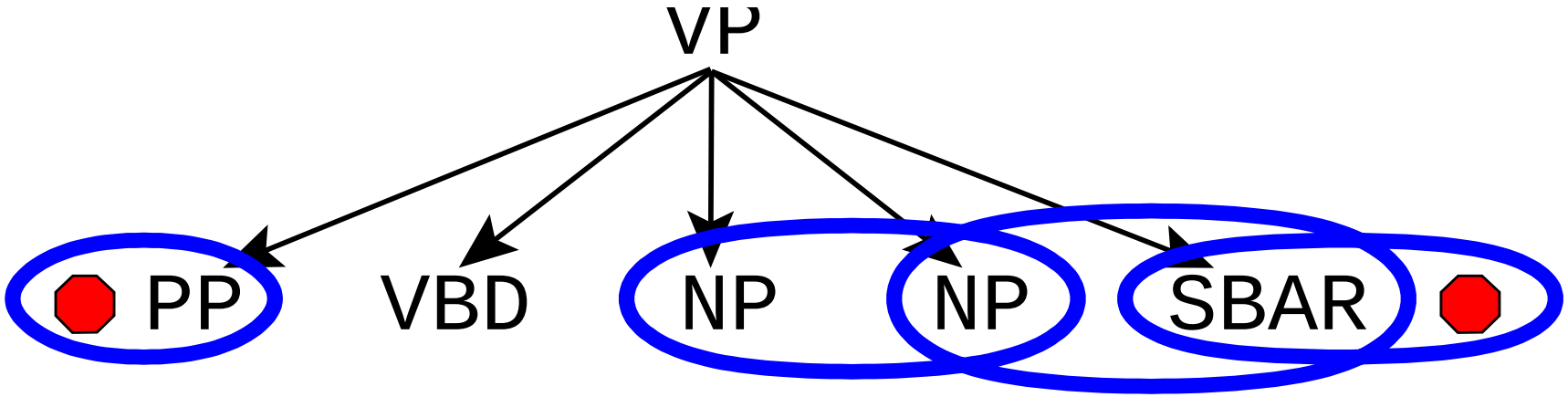
$$
\begin{aligned}
& \mathbf{\Phi}(x, y) \cdot \mathbf{W} \\
= & \sum_i \mathbf{\Phi}_i(x, y) \mathbf{W}_i \\
= & \mathbf{W}_1 \mathbf{\Phi}_1(x, y) + \mathbf{W}_{54} + \mathbf{W}_{118} + \mathbf{W}_{14} + \mathbf{W}_{10078} + \mathbf{W}_{9000} + \mathbf{W}_{1078} + \mathbf{W}_{101}
\end{aligned}
$$

From [Collins and Koo, 2005]:
The following types of features were included in the model. We will use the rule `VP -> PP VBD NP NP SBAR` with head `VBD` as an example. Note that the output of our baseline parser produces syntactic trees with headword annotations.

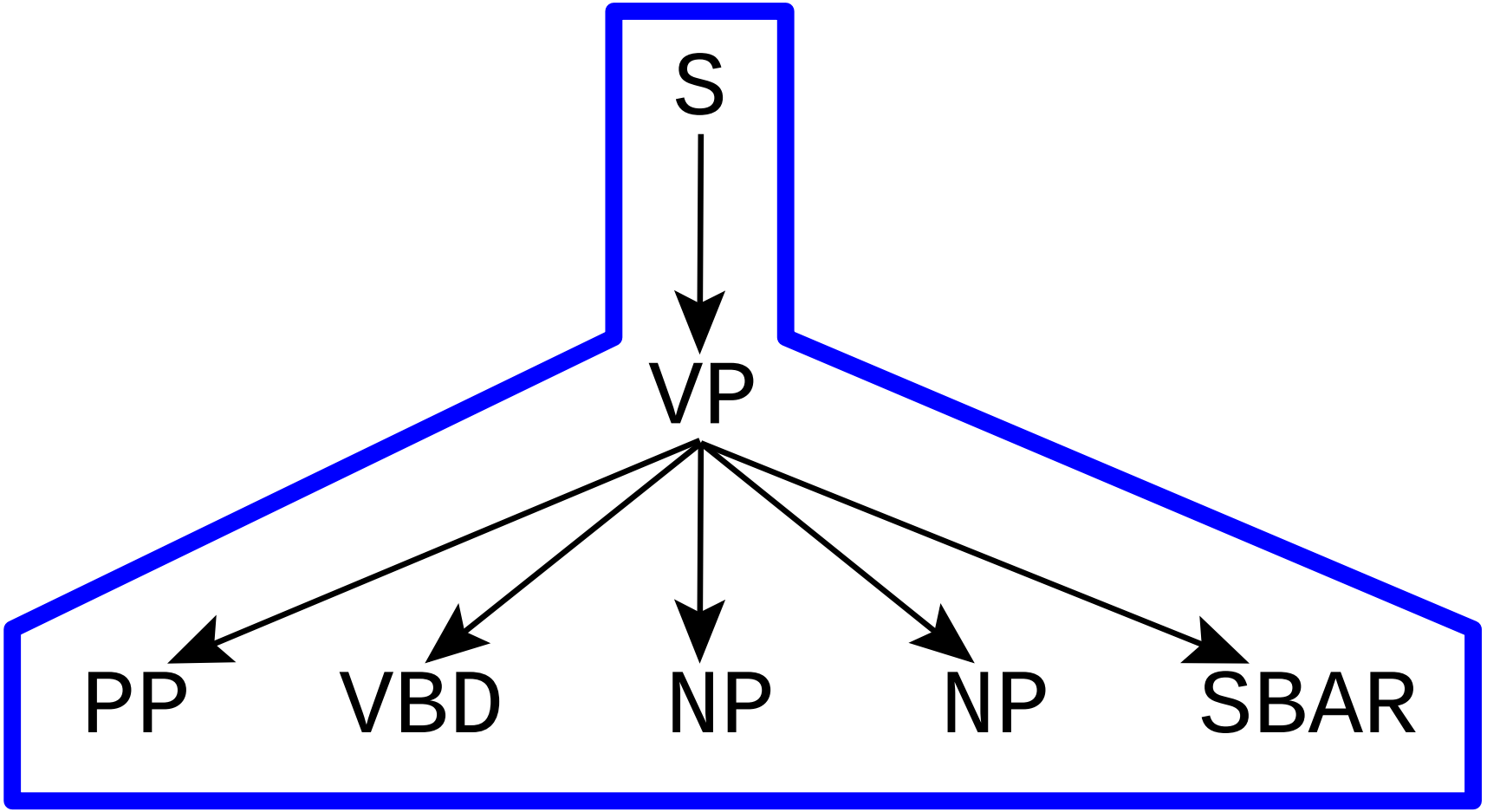
Rules These include all context-free rules in the tree, for example `VP -> PP VBD NP NP SBAR`.



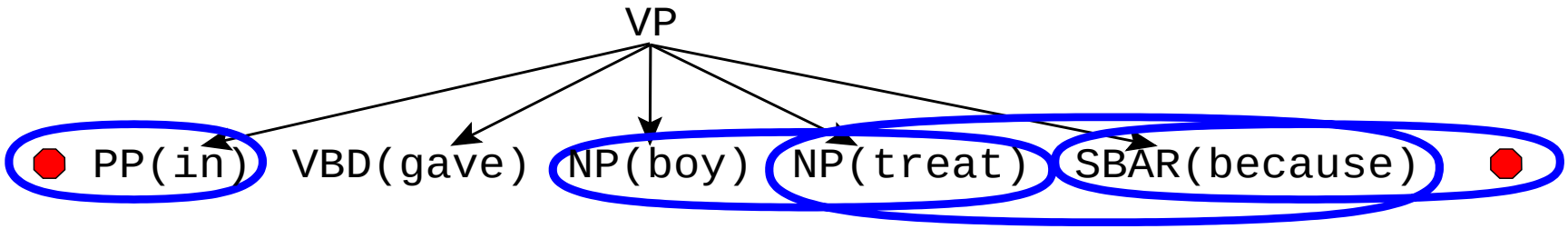
Bigrams These are adjacent pairs of non-terminals to the left and right of the head. As shown, the example rule would contribute the bigrams `(Right,VP,NP,NP)`, `(Right,VP,NP,SBAR)`, `(Right,VP,SBAR,STOP)`, and `(Left,VP,PP,STOP)` to the left of the head.



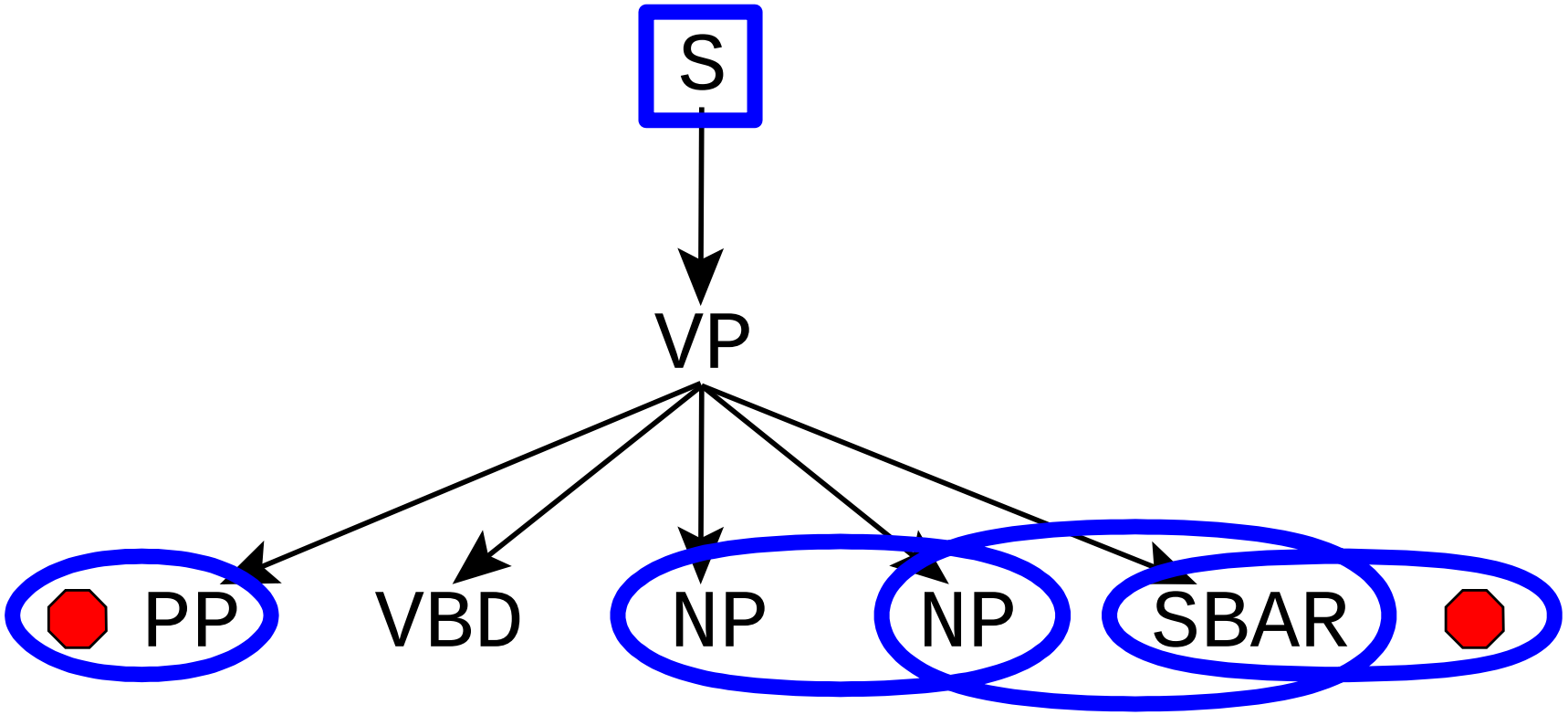
Grandparent Rules Same as **Rules**, but also including the non-terminal above the rule.



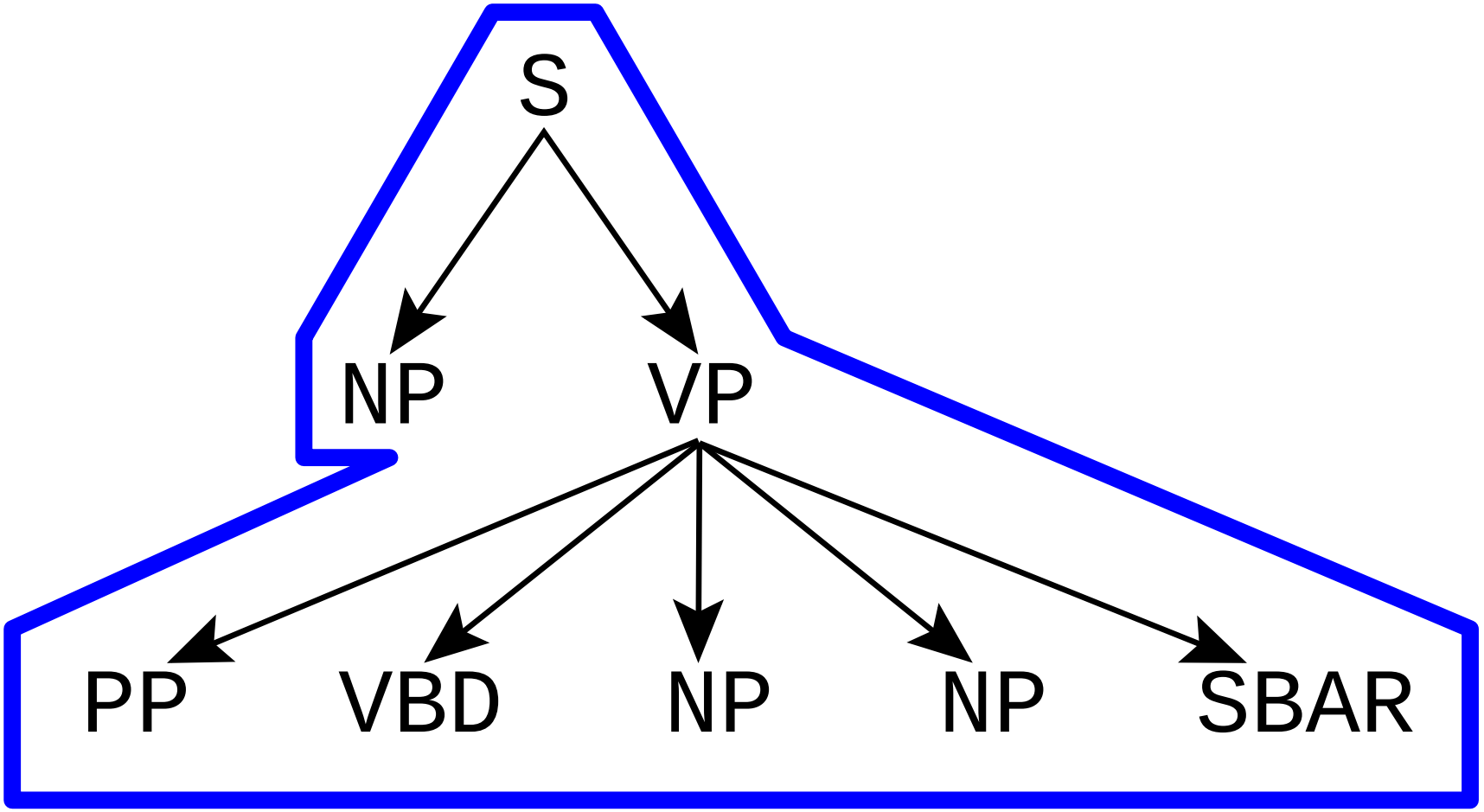
Lexical Bigrams Same as **Bigrams**, but with the lexical heads of the two non-terminals also included.



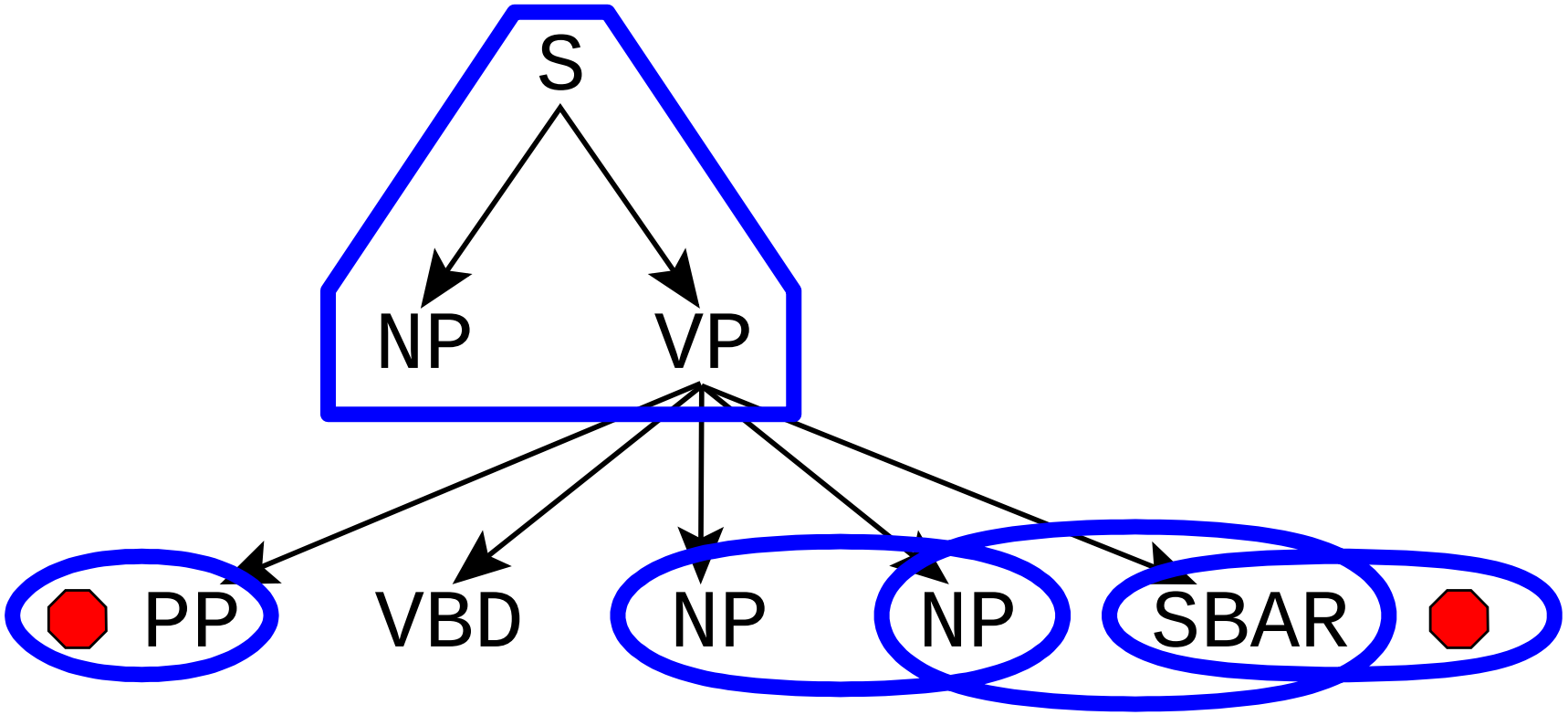
Grandparent Bigrams Same as **Bigrams**, but also including the non-terminal above the bigrams.



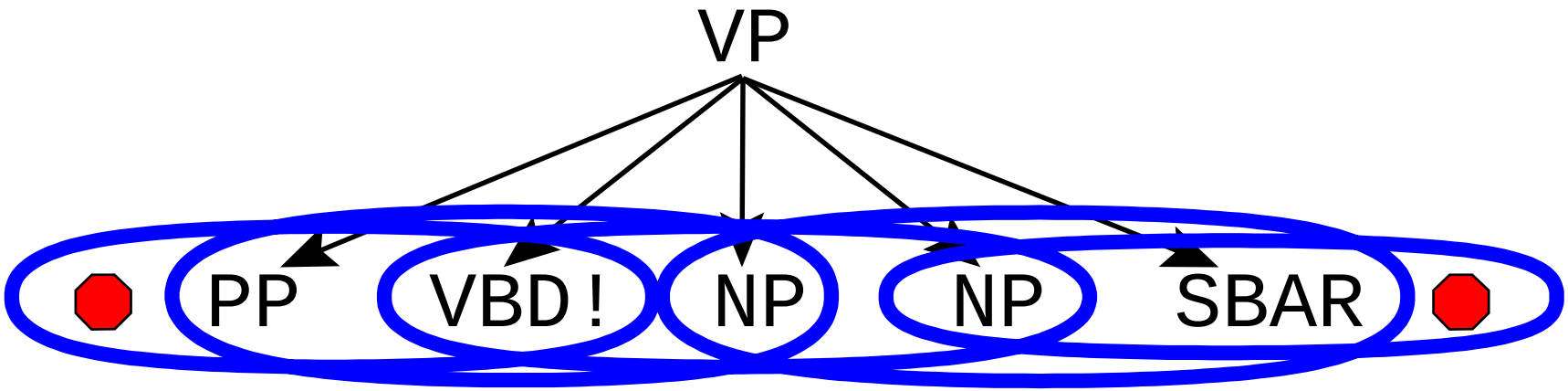
Two-level Rules Same as **Rules**, but also including the entire rule above the rule.



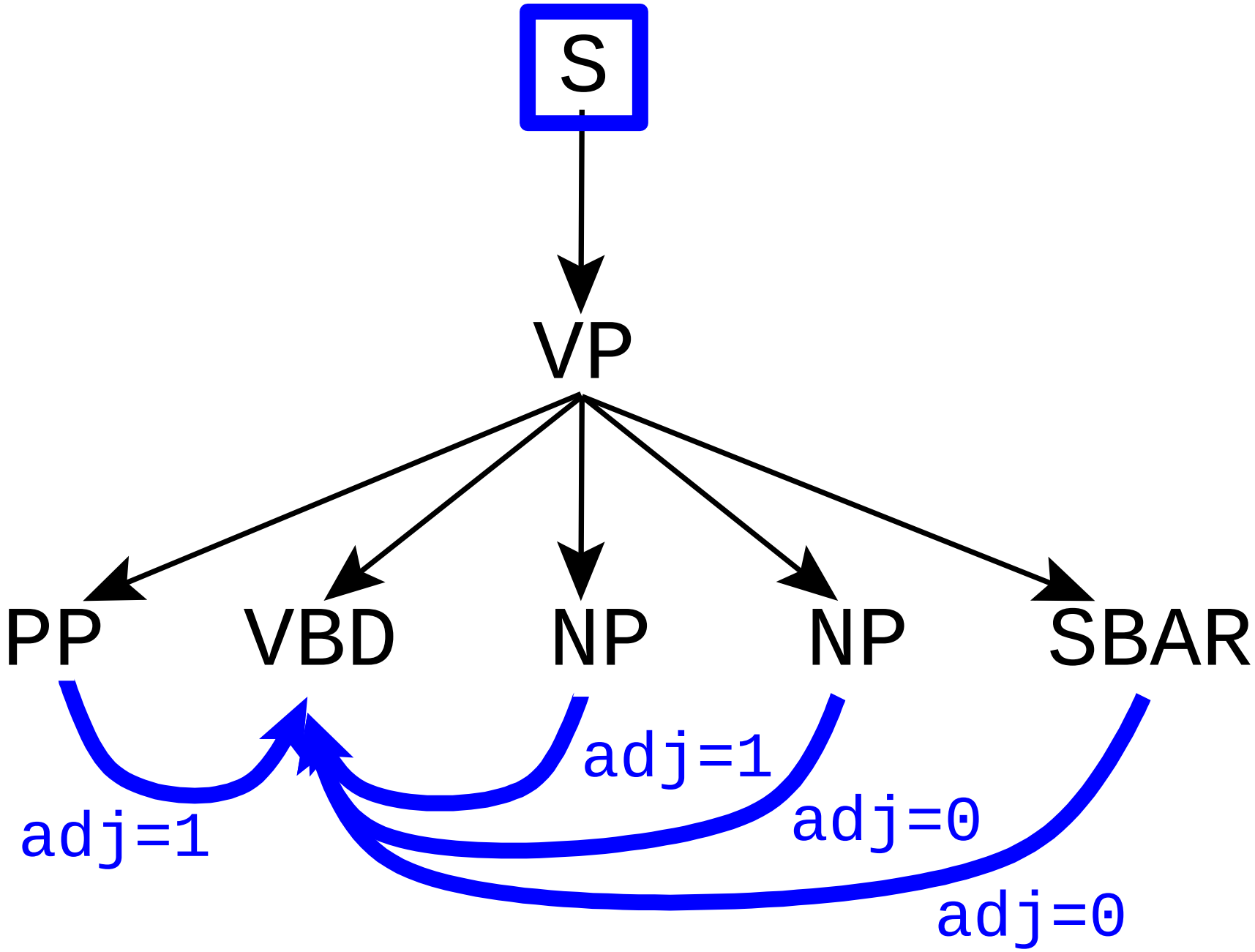
Two-level Bigrams Same as **Bigrams**, but also including the entire rule above the rule.



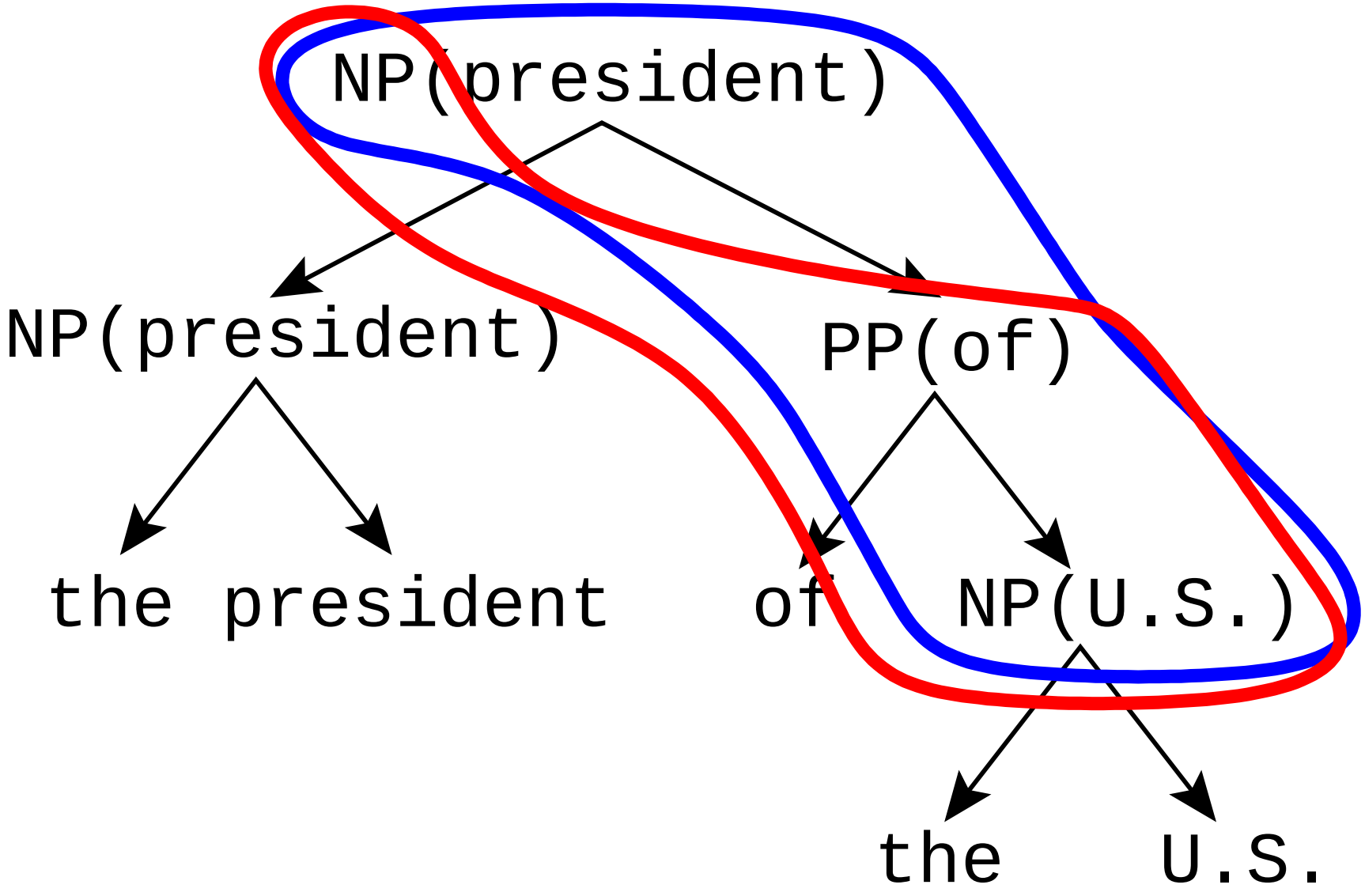
Trigrams All trigrams within the rule. The example rule would contribute the trigrams `(VP,STOP,PP,VBD!)`, `(VP,PP,VBD!,NP)`, `(VP, VBD!,NP,NP)`, `(VP,NP,NP,SBAR)` and `(VP,NP, SBAR,STOP)` (! is used to mark the head of the rule)



Head-Modifiers All head-modifier pairs, with the grandparent non-terminal also included. An `adj` flag is also included, which is 1 if the modifier is adjacent to the head, 0 otherwise. As an example, say the non-terminal dominating the example rule is `S`. The example rule would contribute `(Left,S,VP,VBD,PP,adj=1)`, `(Right,S,VP,VBD,NP,adj=1)`, `(Right,S,VP, VBD,NP,adj=0)`, and `(Right,S,VP,VBD,SBAR, adj=0)`.


S
VP
PP
VBD
NP
NP
SBAR
adj=1
adj=1
adj=0
adj=0

PPs Lexical trigrams involving the heads of arguments of prepositional phrases. The example shown at right would contribute the trigram `(NP,NP,PP,NP,president,of,U.S.)`, in addition to the more general trigram relation `(NP,NP,PP,NP,of,U.S.)`.


NP(president)
NP(president)
PP(of)
the president
of
NP(U.S.)
the
U.S.

Distance Head-Modifiers Features involving the distance between head words. For example, assume $dist$ is the number of words between the head words of the VBD and SBAR in the (VP,VBD,SBAR) head-modifier relation in the above rule. This relation would then generate features (VP,VBD,SBAR,$= dist$), and (VP,VBD,SBAR,$\leq x$) for all $dist \leq x \leq 9$ and (VP,VBD,SBAR,$\geq x$) for all $1 \leq x \leq dist$.

Further Lexicalization In order to generate more features, a second pass was made where all non-terminals were augmented with their lexical heads when these headwords were closed-class words. All features apart from **Head-Modifiers**, **PPs** and **Distance Head-Modifiers** were then generated with these augmented non-terminals.

Overview

- A brief review of history-based methods
- A new framework: Global linear models
- Parsing problems in this framework:
  Reranking problems
- Parameter estimation method 1:
  A variant of the perceptron algorithm

A Variant of the Perceptron Algorithm

Inputs: Training set (x_i, y_i) for $i = 1 \ldots n$

Initialization: $\mathbf{W} = 0$

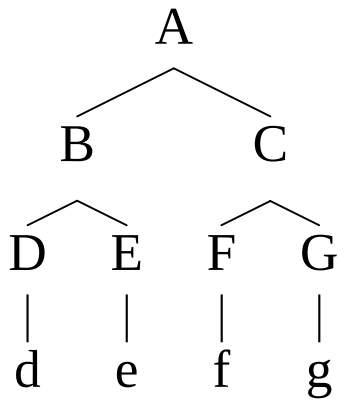
Define: $F(x) = \text{argmax}_{y \in \mathbf{GEN}(x)} \mathbf{\Phi}(x, y) \cdot \mathbf{W}$

Algorithm: For $t = 1 \ldots T$, $i = 1 \ldots n$

$z_i = F(x_i)$

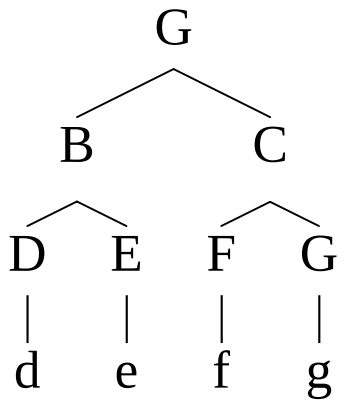
If $(z_i \neq y_i)$ $\mathbf{W} = \mathbf{W} + \mathbf{\Phi}(x_i, y_i) - \mathbf{\Phi}(x_i, z_i)$

Output: Parameters $\mathbf{W}$



$\Rightarrow$

log probability	-1.56
HASRULE:A->B;C	54
HASRULE:B->D;E	118
HASRULE:C->F;G	14
HASRULE:D->d	10078
HASRULE:E->e	9000
HASRULE:F->f	1078
HASRULE:G->g	101



$\Rightarrow$

log probability	-1.13
HASRULE:G->B;C	89
HASRULE:B->D;E	118
HASRULE:C->F;G	14
HASRULE:D->d	10078
HASRULE:E->e	9000
HASRULE:F->f	1078
HASRULE:G->g	101

Say first tree is correct, but second tree has higher $\mathbf{W} \cdot \mathbf{\Phi}(x, y)$ under current parameters: Then $\mathbf{W} = \mathbf{W} + \mathbf{\Phi}(x_i, y_i) - \mathbf{\Phi}(x_i, z_i)$ implies

$$\begin{aligned} \mathbf{W}_1 &+= -1.56 - (-1.13) = -0.43 \\ \mathbf{W}_{54} &+= 1 \\ \mathbf{W}_{89} &+= -1 \end{aligned}$$

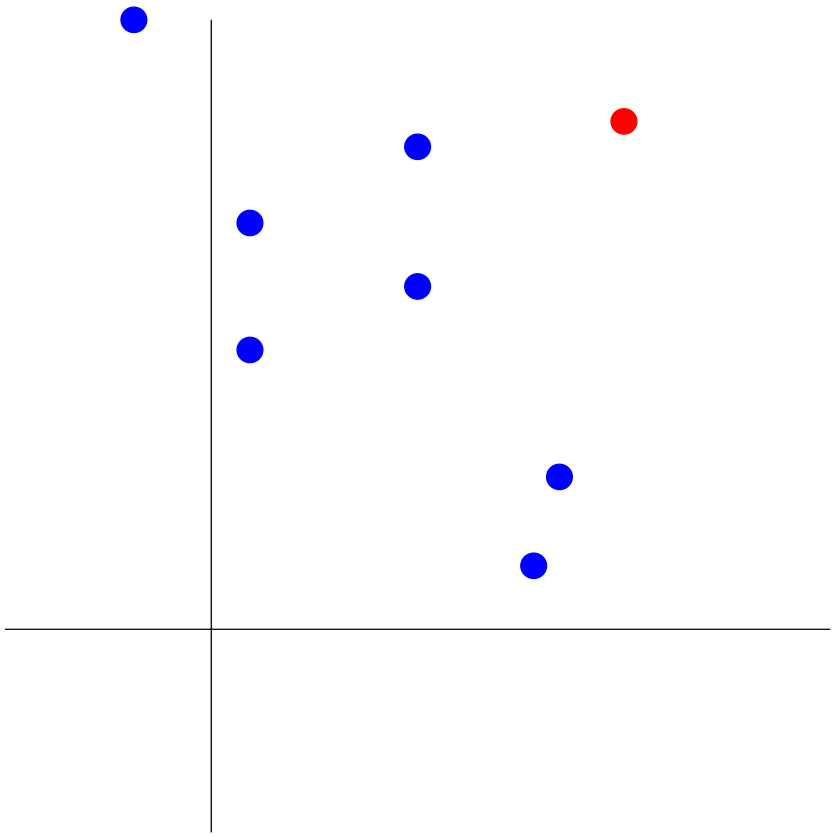
Theory Underlying the Algorithm

- **Definition:** $\overline{\mathbf{GEN}}(x_i) = \mathbf{GEN}(x_i) - \{y_i\}$

- **Definition:** The training set is **separable with margin** δ, if there is a vector $\mathbf{U} \in \mathbb{R}^d$ with $||\mathbf{U}|| = 1$ such that

$$\forall i, \forall z \in \overline{\mathbf{GEN}}(x_i) \quad \mathbf{U} \cdot \mathbf{\Phi}(x_i, y_i) - \mathbf{U} \cdot \mathbf{\Phi}(x_i, z) \geq \delta$$

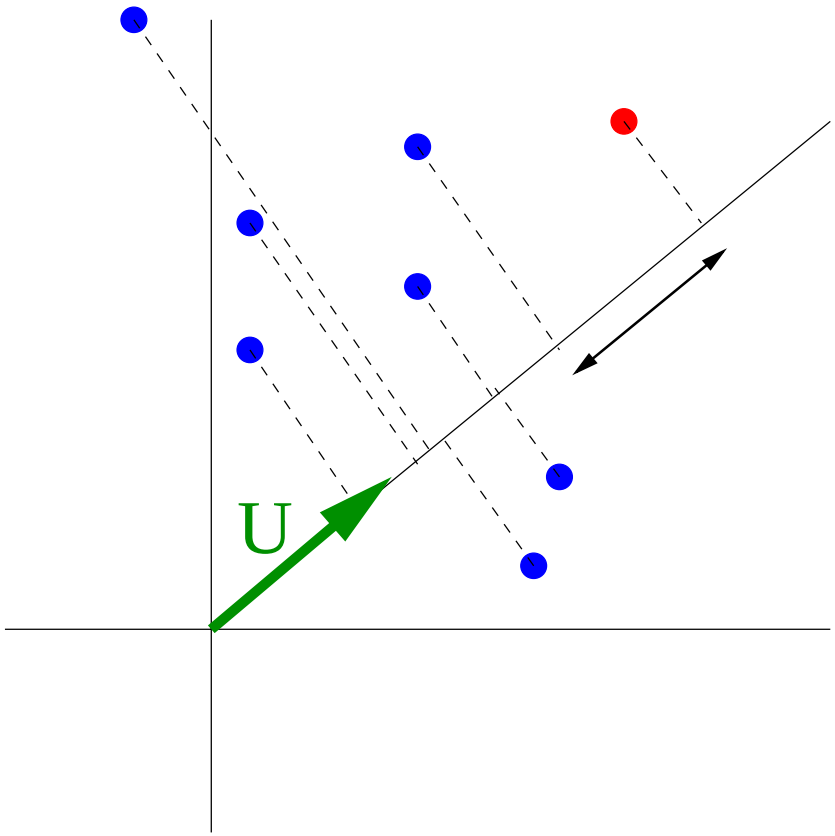
GEOMETRIC INTUITION BEHIND SEPARATION



= Correct candidate

= Incorrect candidates

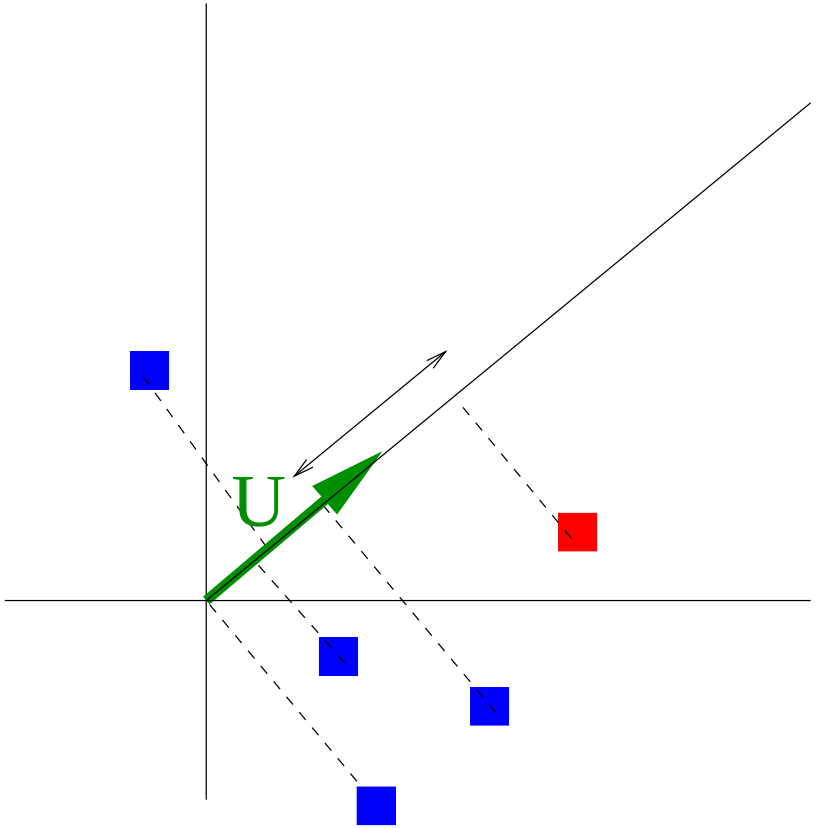
GEOMETRIC INTUITION BEHIND SEPARATION



= Correct candidate

= Incorrect candidates

ALL EXAMPLES ARE SEPARATED



= Correct candidate (2)

= Incorrect candidates (2)

THEORY UNDERLYING THE ALGORITHM

Theorem: For any training sequence (x_i, y_i) which is separable with margin δ, then for the perceptron algorithm

$$\text{Number of mistakes} \leq \frac{R^2}{\delta^2}$$

where R is a constant such that $\forall i, \forall z \in \overline{\mathbf{GEN}}(x_i)$
$||\mathbf{\Phi}(x_i, y_i) - \mathbf{\Phi}(x_i, z)|| \leq R$

Proof: Direct modifi cation of the proof for the classifi cation case.

Proof:

Let $\mathbf{W}^k$ be the weights before the k'th mistake. $\mathbf{W}^1 = 0$
If the k'th mistake is made at i'th example,
and $z_i = \text{argmax}_{y \in \mathbf{GEN}(x_i)} \mathbf{\Phi}(y) \cdot \mathbf{W}^k$, then

$$\begin{aligned}
\mathbf{W}^{k+1} &= \mathbf{W}^k + \mathbf{\Phi}(y_i) - \mathbf{\Phi}(z_i) \\
\Rightarrow \mathbf{U} \cdot \mathbf{W}^{k+1} &= \mathbf{U} \cdot \mathbf{W}^k + \mathbf{U} \cdot \mathbf{\Phi}(y_i) - \mathbf{U} \cdot \mathbf{\Phi}(z_i) \\
&\geq \mathbf{U} \cdot \mathbf{W}^k + \delta \\
&\geq k\delta \\
\Rightarrow ||\mathbf{W}^{k+1}|| &\geq k\delta
\end{aligned}$$

Also,

$$\begin{aligned}
||\mathbf{W}^{k+1}||^2 &= ||\mathbf{W}^k||^2 + ||\mathbf{\Phi}(y_i) - \mathbf{\Phi}(z_i)||^2 + 2\mathbf{W}^k \cdot (\mathbf{\Phi}(y_i) - \mathbf{\Phi}(z_i)) \\
&\leq ||\mathbf{W}^k||^2 + R^2 \\
\Rightarrow ||\mathbf{W}^{k+1}||^2 &\leq kR^2 \\
\Rightarrow k^2\delta^2 &\leq ||\mathbf{W}^{k+1}||^2 \leq kR^2 \\
\Rightarrow k &\leq R^2/\delta^2
\end{aligned}$$

Perceptron Experiments: Parse Reranking

Parsing the Wall Street Journal Treebank

Training set = 40,000 sentences, test = 2,416 sentences
Generative model (Collins 1999): 88.2% F-measure
Reranked model: 89.5% F-measure (11% relative error reduction)
Boosting: 89.7% F-measure (13% relative error reduction)

- Results from Charniak and Johnson, 2005:
  - Improvement from 89.7% (baseline generative model) to 91.0% accuracy
  - Uses a log-linear model
  - Gains from improved n-best lists, better features

Summary

- A new framework: **global linear models**
  GEN, **Φ**, **W**
- There are several ways to train the parameters **W**:
  - Perceptron
  - Boosting
  - Log-linear models (maximum-likelihood)
- Applications:
  - Reranking models
  - LFG parsing
  - Generation
  - Machine translation
  - Tagging problems
  - Speech recognition