Visual Interpretation using Probabilistic Grammars

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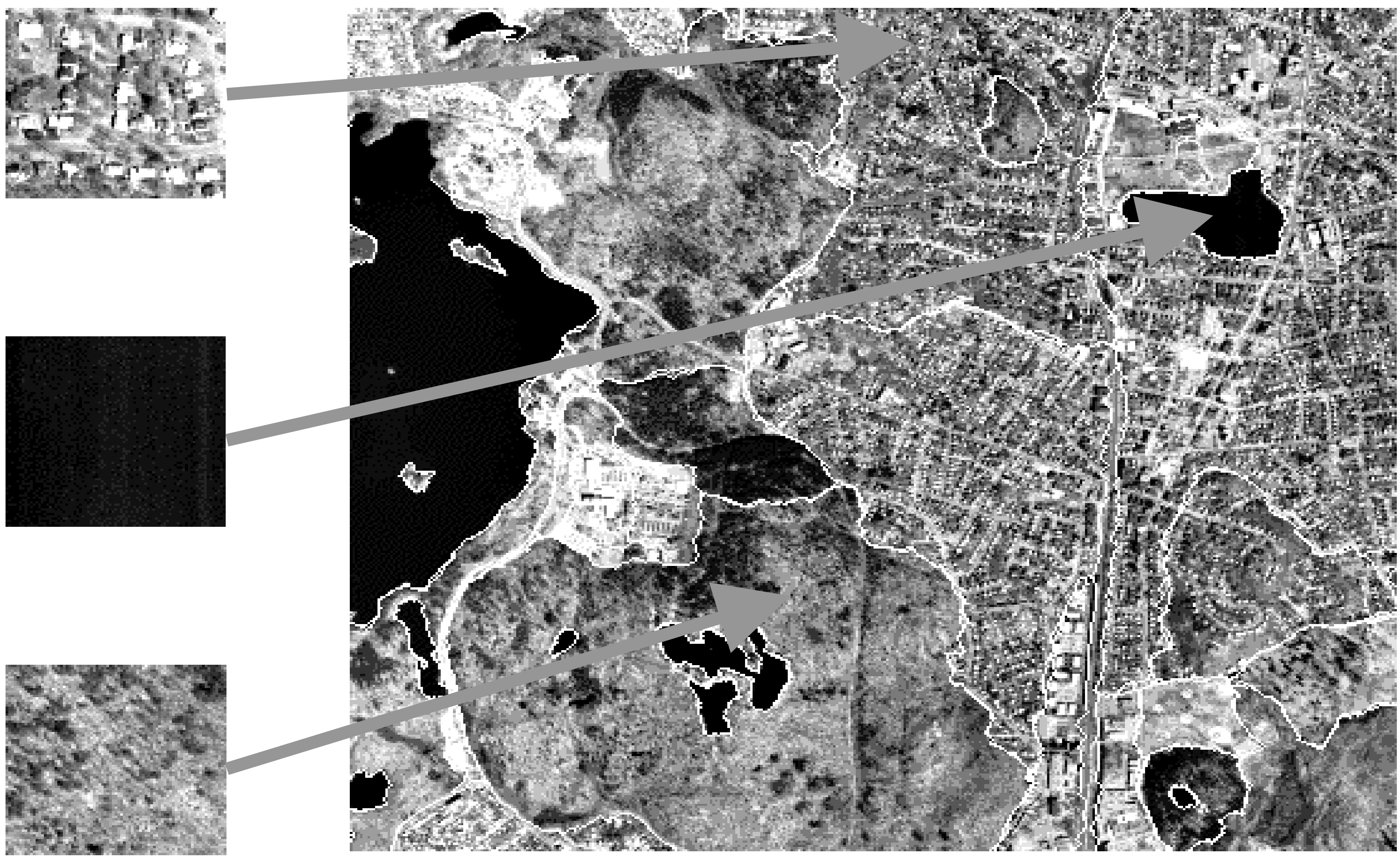
Model-Based Vision

- What do the models look like
- Where do the models come from
- How are the models utilized

The Problem



Optimization/Search Problem

Find the most likely interpretation of the image contents that:

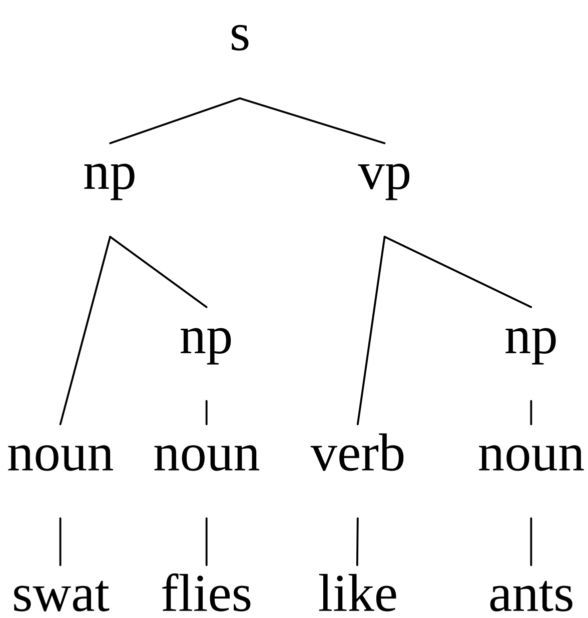
1. Identifies the component parts of the image correctly.
2. Identifies the scene type.
3. Identifies structural relationships between the parts of the image.

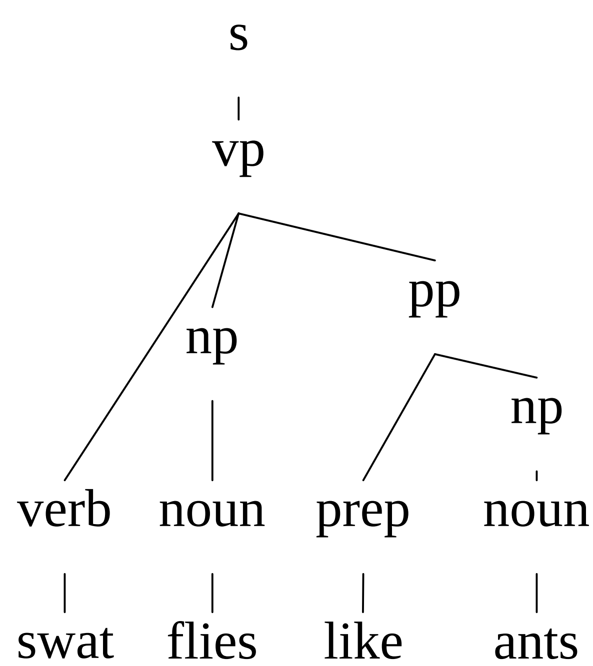
Involves: Segmenting into parts, naming the parts, and relating the parts.

Outline

- Overview of statistical methods used in speech recognition and NLP
- Image Segmentation and Interpretation
  - image grammars
  - image grammar learning
  - algorithms for parsing patchwork images.

Not any description – the best





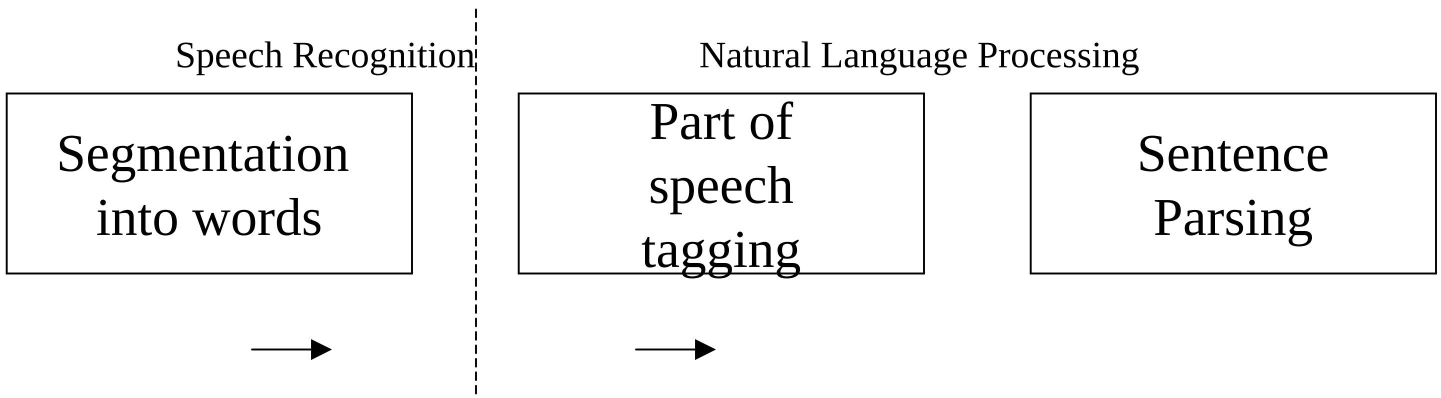
Bad parse

Good parse

What’s similar/different between image analysis and speech recognition/NLP?

- Similar
  - An input signal must be processed.
  - Segmentation.
  - Identification of components.
  - Structural understanding.
- Dissimilar
  - Text is a valid intermediate goal that separates Speech recognition and NLP. Line drawings are less obviously useful.
  - Structure in images has much more richness.

Speech Recognition and NLP



- Little backward flow
- Stages done separately.
- Similar techniques work well in each of these phases.
- A parallel view can also be applied to image analysis.

Speech Understanding

- Goal: Translate the input signal into a sequence of words.
  - Segment the signal into a sequence of samples.
    - $A = a_1, a_2, ..., a_m \quad a_i \in \mathcal{A}$
  - Find the best words that correspond to the samples based on:
    - An acoustic model.
      - Signal Processing
      - Prototype storage and comparator (identification)
    - A language model.
    - $W = w_1, w_2, ..., w_m \quad w_i \in \mathcal{V}$
  - $W_{opt} = \arg\max_w P(W|A)$
  - $W_{opt} = \arg\max_w P(A|W)\, P(W)$
    - (since $P(W|A) = P(A|W)\, P(W) / P(A)$ [Bayes])
    - $P(A|W)$ is the acoustic model.
    - $P(W)$ is the language model.

language modeling for speech

$$P(W) = \prod_{i=1}^{n} P(w_i \mid w_1, \ldots, w_{i-1})$$

$$P(W) = \prod_{i=1}^{n} P(w_i \mid \Phi(w_1, \ldots, w_{i-1}))$$

$$P(W) = \prod_{i=1}^{n} P(w_i \mid \Phi_{i-1})$$

$$P(w_i \mid w_{i-1}, w_{i-2}) = f(w_i \mid w_{i-1}, w_{i-2})$$

$$P(w_i \mid w_{i-1}, w_{i-2}) = \lambda_3 f(w_i \mid w_{i-1}, w_{i-2}) + \lambda_2 f(w_i \mid w_{i-1}) + \lambda_1 f(w_i)$$

$$\lambda_1 + \lambda_2 + \lambda_3 = 1$$

- Using the above
  - P(W) can be represented as a HMM and solved efficiently using the Viterbi algorithm.
  - The good weights λ_1, λ_2, and λ_3 can be computed using the Baum-Welch algorithm.

Natural Language Processing

- Part of correctly *understanding* a sentence comes from correctly *parsing* it.
- Starting with a word list, parsing involves two separable activities:
  - Part of speech tagging.
    - Find the most *probable* assignments of parts of speech.
  - Parsing the words into a tree.
    - Find the most *probable* parse tree.

Part-of-speech tagging

- Goal: Assign part-of-speech tags to each word in the word sequence.
  - Start with the word sequence
    - $W = w_1, w_2, \ldots, w_m \quad w_i \in \mathcal{V}$
  - Find the best tags for each word
    - $T = t_1, t_2, \ldots, t_m \quad t_i \in \mathcal{T}$

$$P(w_{1,n}) = \sum_{t_{1,n+1}} P(w_{1,n}, t_{1,n+1})$$

$$T_{opt} = \arg\max_{t_{1,n}} P(t_{1,n} \mid w_{1,n})$$

$$T_{opt} = \arg\max_{t_{1,n}} P(t_{1,n}, w_{1,n})$$

$$P(w_n \mid w_{1,n-1}, t_{1,n}) = P(w_n \mid t_n)$$

$$P(t_n \mid w_{1,n-1}, t_{1,n-1}) = P(t_n \mid t_{n-1})$$

$$P(w_{1,n}) = \sum_{t_{1,n+1}} \prod_{i=1}^{n} P(w_i \mid t_i) P(t_{i+1} \mid t_i)$$

$$P(w1,n) = \sum_{t_{1,n+1}} \prod_{i=1}^{n} P(w_i \mid t_i) P(t_{i+1} \mid t_i, t_{i-1})$$

- T_{opt} is the path the HMM traverses in producing the output (since the states of the HMM are the tags).
- Use Viterbi algorithm to find the path.

PCFG's

- Better language models lead to better results.
- Considering the grammar instead of a simple sequence of words, the relationships are more meaningful.
- PCFG is <W, N, N^1, R>
  - W is a set of terminal symbols
  - N is a set of non-terminal symbols
  - N^1 is the starting symbol
  - R is a set of rules.
    - Each rule $N^i \rightarrow RHS$ has an associated probability $P(N^i \rightarrow RHS)$ which is the probability of using this rule to expand N^i
- The probability of a sentence is the sum of the probabilities of all parses.
- Probability of a parse is the product of the probabilities of all the productions used.
- Smoothing necessary for missing rules.

Example PCFG

s	$\rightarrow$	np vp	0.8
s	$\rightarrow$	vp	0.2
np	$\rightarrow$	noun	0.4
np	$\rightarrow$	noun pp	0.4
np	$\rightarrow$	noun np	0.2
vp	$\rightarrow$	np vp	0.3
vp	$\rightarrow$	np vp	0.3
vp	$\rightarrow$	np vp	0.2
vp	$\rightarrow$	np vp	0.2
pp	$\rightarrow$	prep np	1.0
prep	$\rightarrow$	like	1.0
verb	$\rightarrow$	swat	0.2
verb	$\rightarrow$	flies	0.4
verb	$\rightarrow$	like	0.4
noun	$\rightarrow$	swat	0.1
noun	$\rightarrow$	flies	0.4
noun	$\rightarrow$	ants	0.5

- Good parse = .2x.2x.2x.4x.4x1.0x1.0x.4x.5 = 0.000256
- Bad parse =.8x.2x.4x.1x.4x.3x.4x.4x.5 = 0.00006144

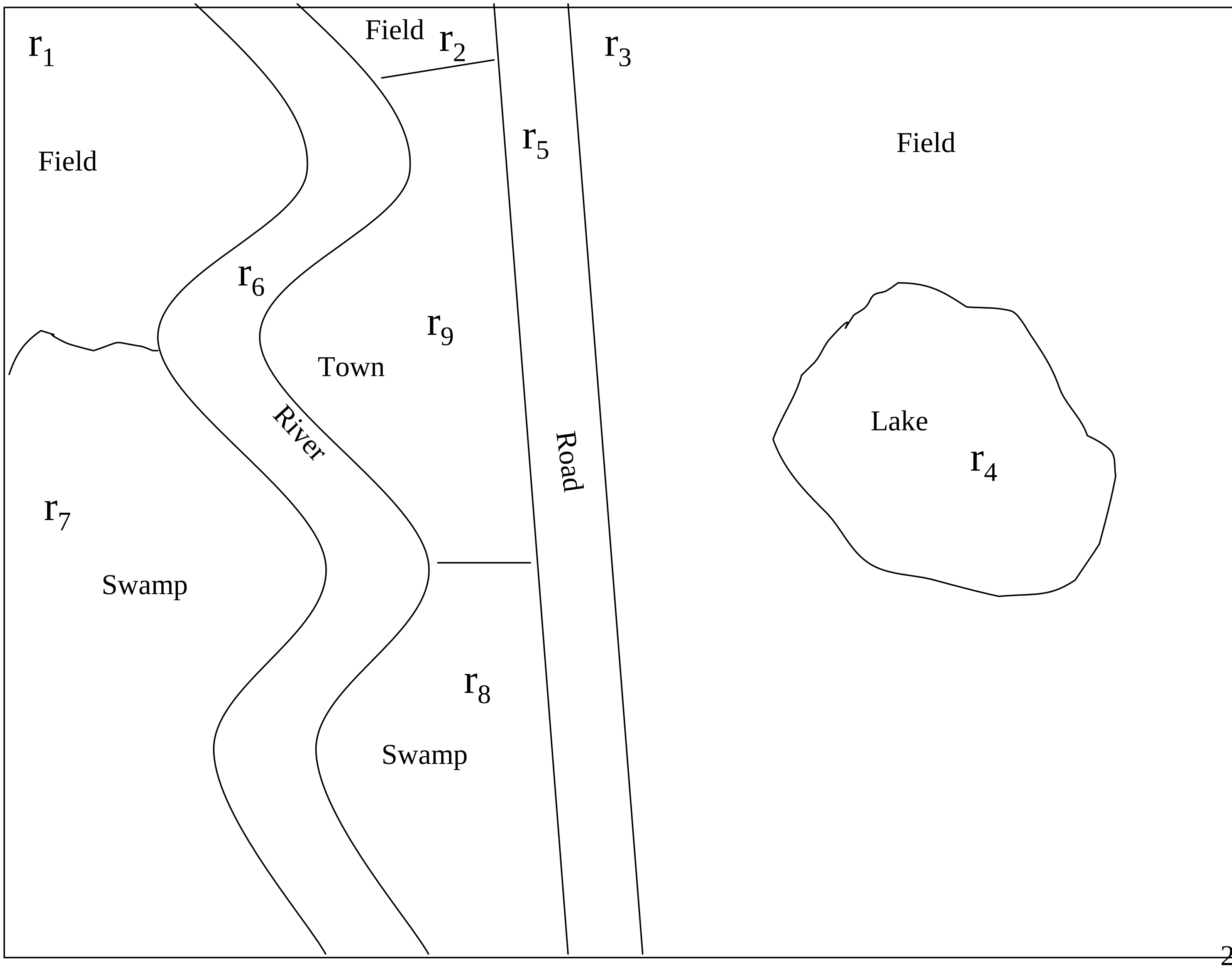
Why these techniques are dominating language research

- Statistical methods work well
  - The best POS taggers perform close to 97% accuracy compared to human accuracy of 98%.
  - The best statistical parsers are at around 88% vs an estimated 95% for humans.
- Learning from the corpus
  - The grammar can be learned from a representative corpus.
- Basis for comparison
  - The availability of corpora with ground truth enables researchers to compare their performance against other published algorithms/models.
- Performance
  - Most algorithms at runtime are fast.

Build Image Descriptions

Patchwork Parsing

- Use semantic segmentation to produce a set of homogeneous regions
- Based on the contents of the regions and their shape hypothesize region contents.
- Region contents is ambiguous in isolation
  - Use contextual information to reduce ambiguity.
- The image must make sense
  - We must be able to produce a parse for it.
- Our interpretation of the image approximates the *most probable parse.*
  - Success of the picture language model determines whether most-probable-parse works.
- Do it (nearly) as well as human experts


r_1
Field
Field
r_2
r_3
r_5
Field
r_6
r_9
Town
River
Road
Lake
r_4
r_7
Swamp
r_8
Swamp

Segmented image labeling

- The image contains n regions $r_{1,n}$.
- Each region has a set of neighbors $n_{1,n}$.
- $P(r_{1,n})$ is the sum of the disjoint labelings.

$$P(r_{1,n}) = \sum_{l_{1,n}} P(r_{1,n}, l_{1,n})$$

- We wish to find the labeling $L_{1,n}$.

$$L_{1,n} = \arg\max_{l_{1,n}} \prod_{i=1}^{n} P(l_i \mid r_i, n_i)$$

$$= \arg\max_{l_{1,n}} \prod_{i=1}^{n} \frac{P(l_i \mid r_i) P(n_i \mid l_i, r_i)}{P(n_i \mid r_i)}$$

$$= \arg\max_{l_{1,n}} \prod_{i=1}^{n} \frac{P(l_i \mid r_i) P(n_i \mid l_i)}{P(n_i \mid r_i)}$$

$$= \arg\max_{l_{1,n}} \prod_{i=1}^{n} P(l_i \mid r_i) P(n_i \mid l_i)$$

- $P(l_i|r_i)$ is the optical model.
- $P(n_i|l_i)$ is the picture language model.

Segmentation

The optical model

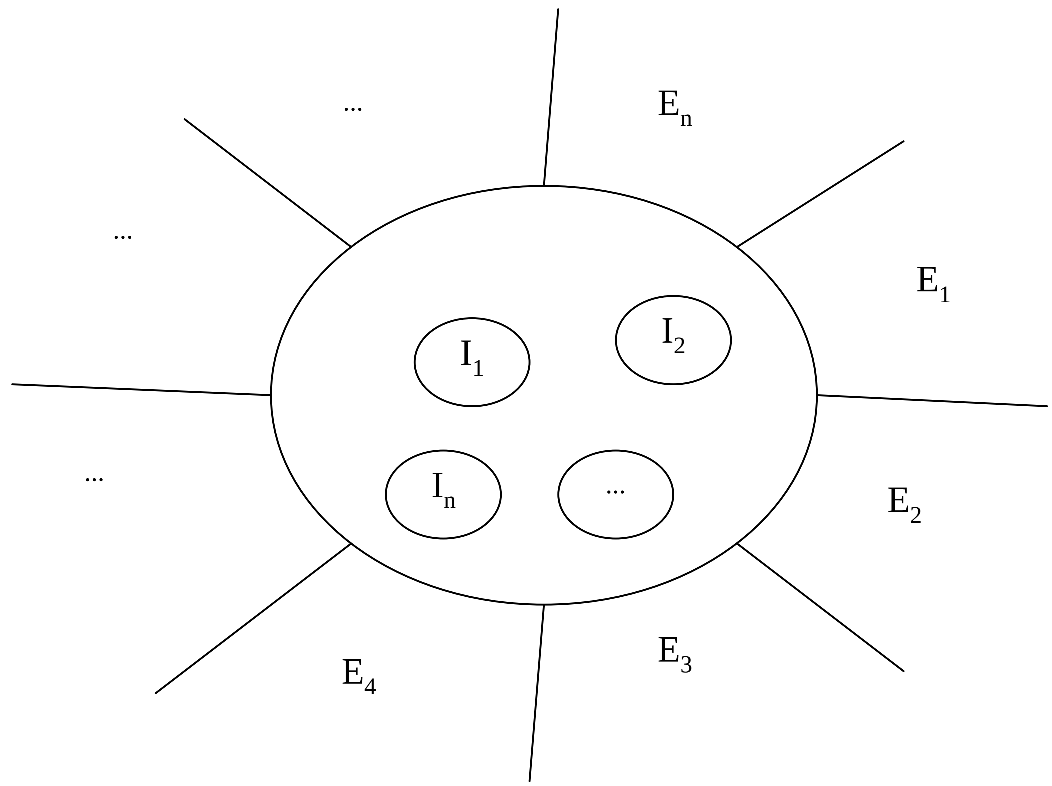
- Filters produce useful features from the original image.
- Semantic Segmentation produces regions.
- Prototype database and comparator produce evidence for labeling each region.

```
(setq *region-optical-evidence*
  '((r1 (field . .5) (swamp . .2) (town . .1) (lake . .1) (road . .05) (river . .05))
    (r2 (field . .5) (swamp . .2) (town . .1) (lake . .1) (road . .05) (river . .05))
    (r3 (field . .5) (swamp . .2) (town . .1) (lake . .1) (road . .05) (river . .05))
    (r4 (field . .1) (swamp . .1) (town . .1) (lake . .3) (road . .1)  (river . .3))
    (r5 (field . .1) (swamp . .1) (town . .3) (lake . .1) (road . .3)  (river . .1))
    (r6 (field . .1) (swamp . .1) (town . .1) (lake . .3) (road . .1)  (river . .3))
    (r7 (field . .3) (swamp . .4) (town . .1) (lake . .1) (road . .05) (river . .05))
    (r8 (field . .3) (swamp . .4) (town . .1) (lake . .1) (road . .05) (river . .05))
    (r9 (field . .1) (swamp . .2) (town . .5) (lake . .1) (road . .05) (river . .05))
  ))
```

$$R = \{< r_1, \{< l_1, P(l_1 \mid r_1) >, \ldots\} >, \ldots\}$$

$$\forall r_i \in R : \sum_{j=1}^{n} P(l_j \mid r_i) \leq 1$$

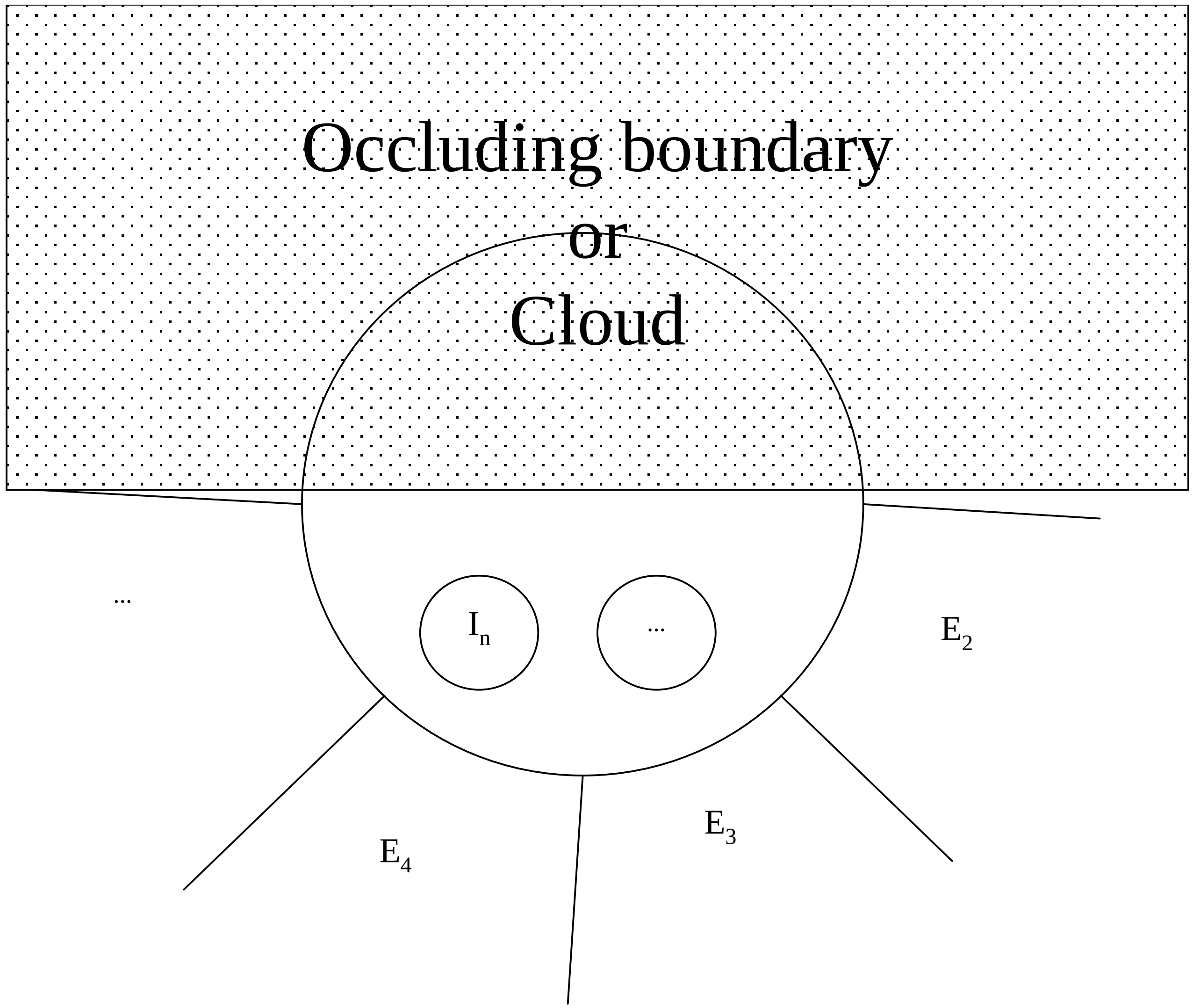
Language Model



- Regions have internal and external neighbors.
- Rule for a region looks this:

<Label, Internal, External, Probability>

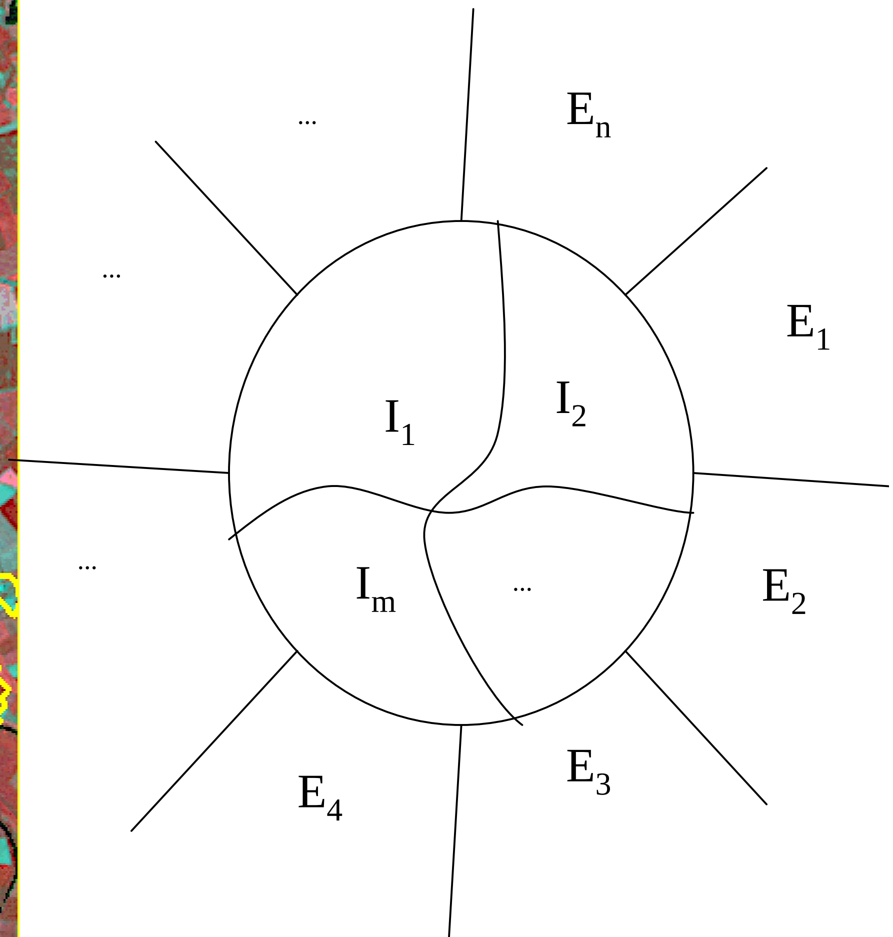
<Field, $(I_1, I_2, \ldots I_n)$, $(E_1, E_2, E_3, E_4, \ldots E_n)$, 0.3>



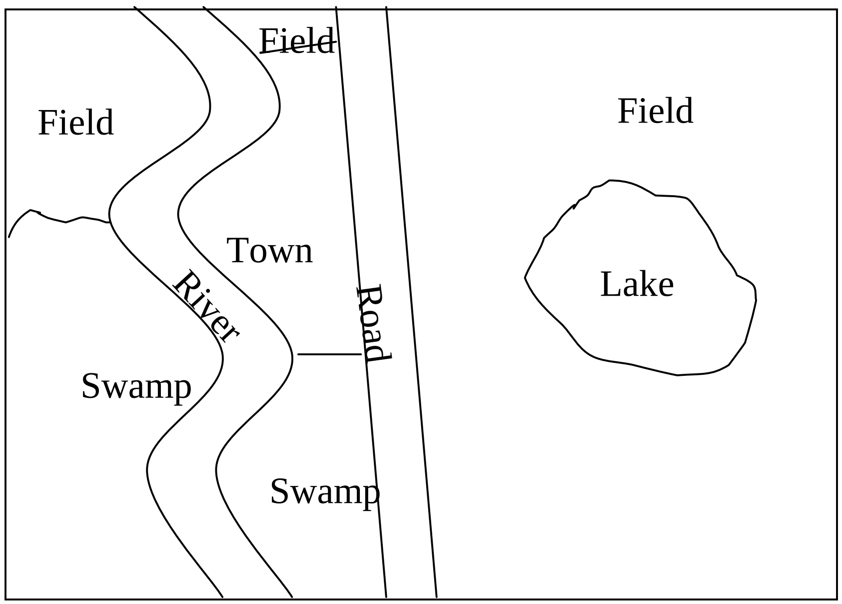
- Regions may be occluded.
- Rule for a region looks this:

<Field, (*, I_n), (*, E_2, E_3, E_4,... E_n), 0.3>

Structured Regions



Example rules



- P_1: <lake, (), (field), 1.0>
- P_2: <field, (lake, *), (road *), 0.33>
- P_3: <field, (*), (*, road, town, river), 0.33>
- P_4: <field, (*), (*, river, swamp), 0.33>
- P_5: <swamp, (*), (* field river), 0.5>
- P_6: <swamp, (*), (* river town road), 0.5>
- P_7: <river, (*), (* field town swamp * swamp field), 1.0>
- p_8: <town, (), (field road swamp river), 1.0>

Supervised Learning

Smoothing and occlusion

- Whenever we generate a rule, we also make rules for degenerate cases.

  <Field, (), (E_1, E_2, E_3), p?>

  <Field, (), (*, E_2, E_3), p?>

  <Field, (), (E_1, *, E_3), p?>

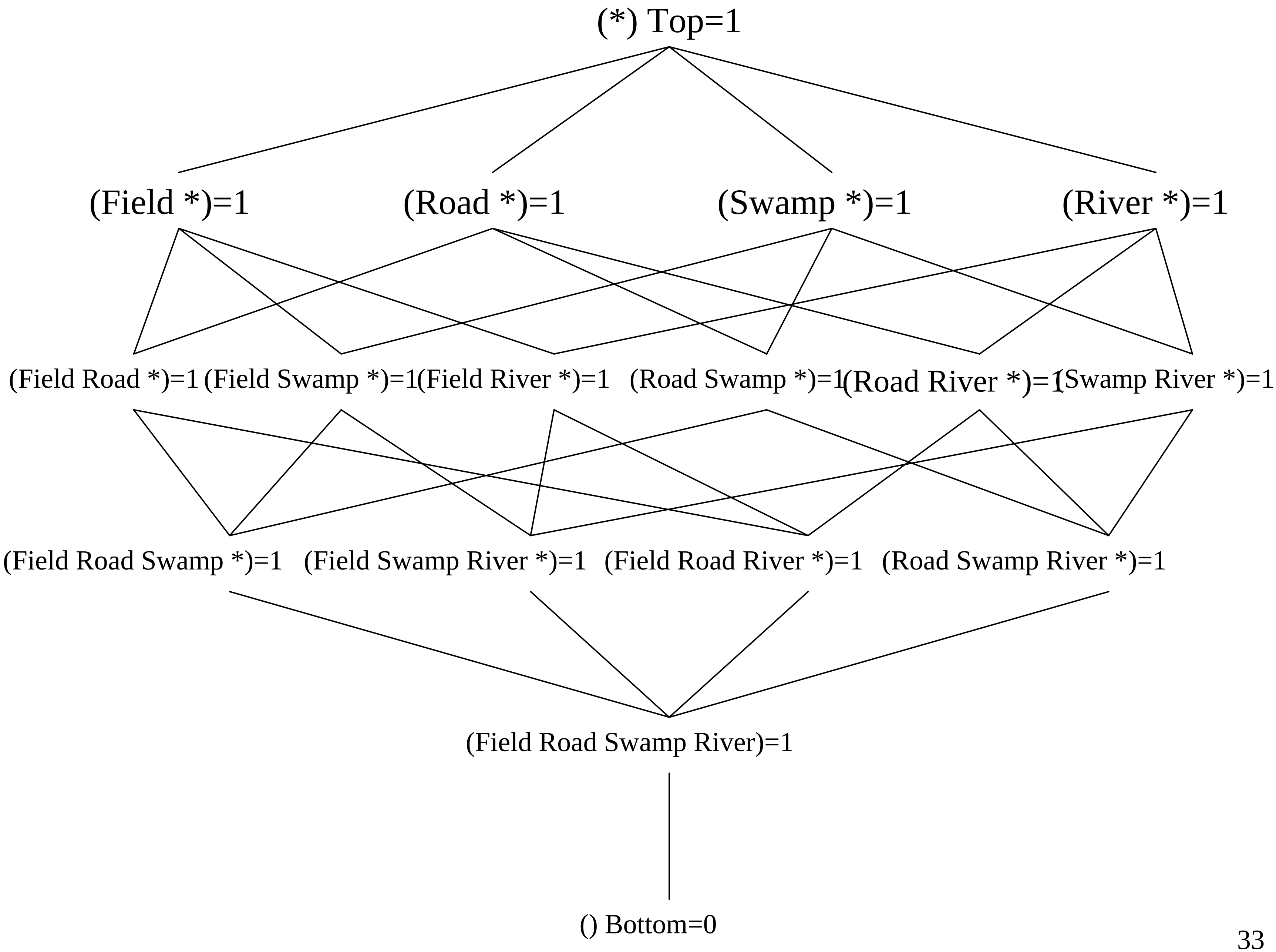
  <Field, (), (E_1, E_2, *), p?>

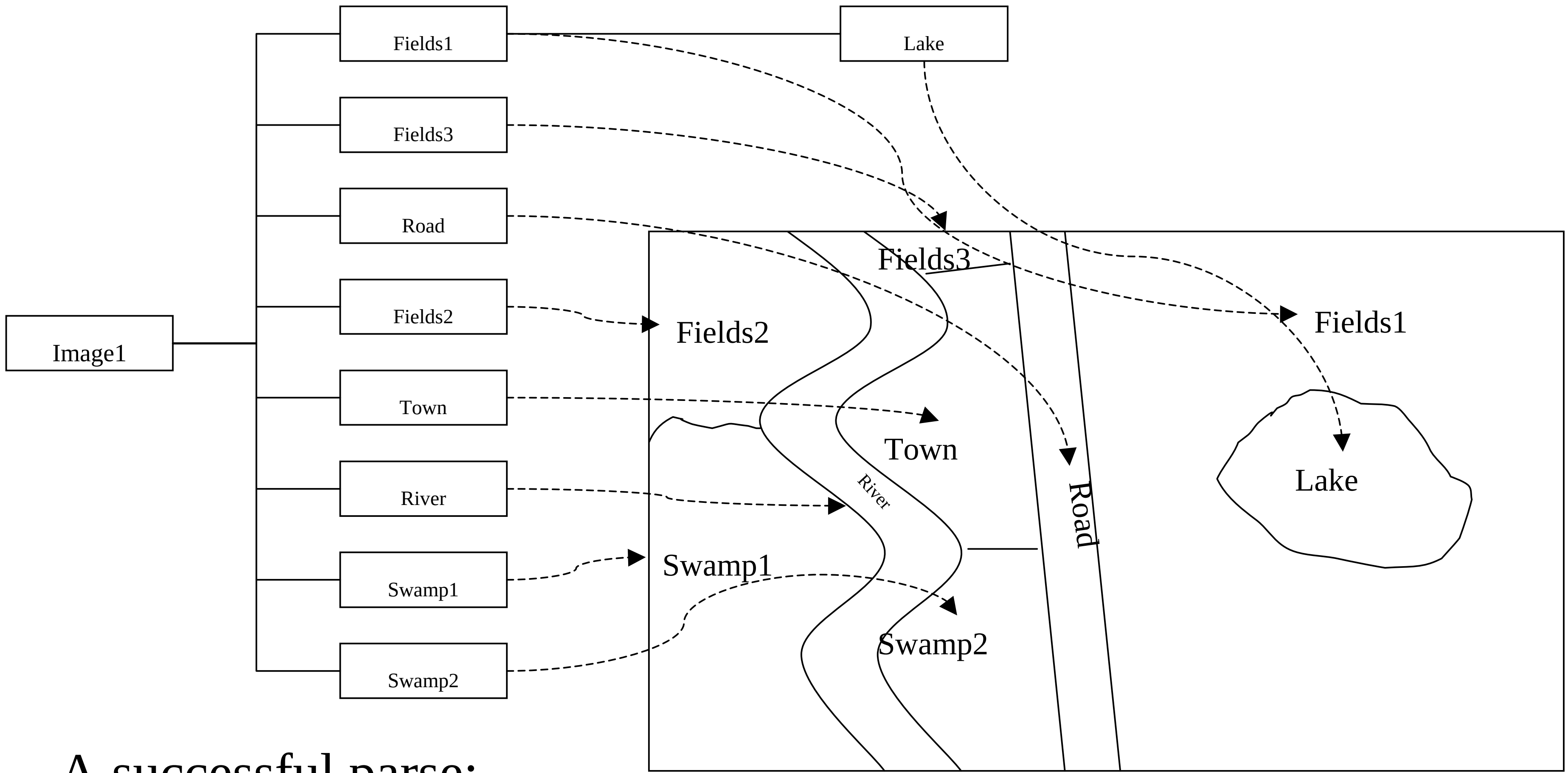
  <Field, (), (*, E_3), p?>

  <Field, (), (*, E_2), p?>

  <Field, (), (*, E_1), p?>

- Represent grammar as a lattice of approximations to the non-occluded rule.


(*) Top=1
(Field *)=1
(Road *)=1
(Swamp *)=1
(River *)=1
(Field Road *)=1
(Field Swamp *)=1
(Field River *)=1
(Road Swamp *)=1
(Road River *)=1
(Swamp River *)=1
(Field Road Swamp *)=1
(Field Swamp River *)=1
(Field Road River *)=1
(Road Swamp River *)=1
(Field Road Swamp River)=1
() Bottom=0



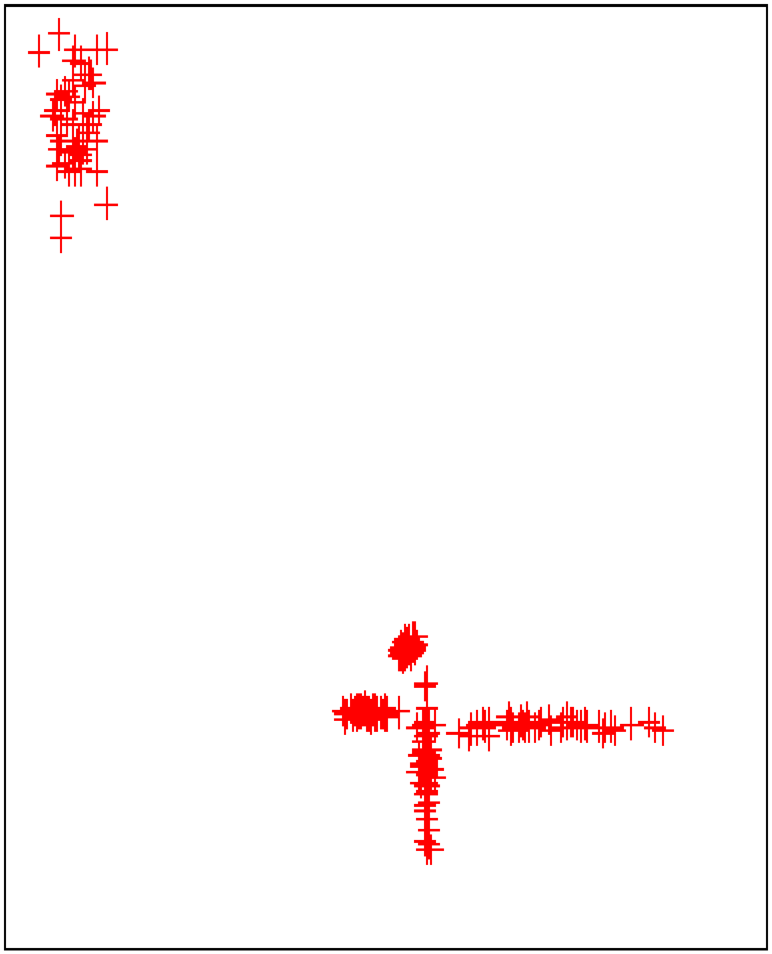
A successful parse:

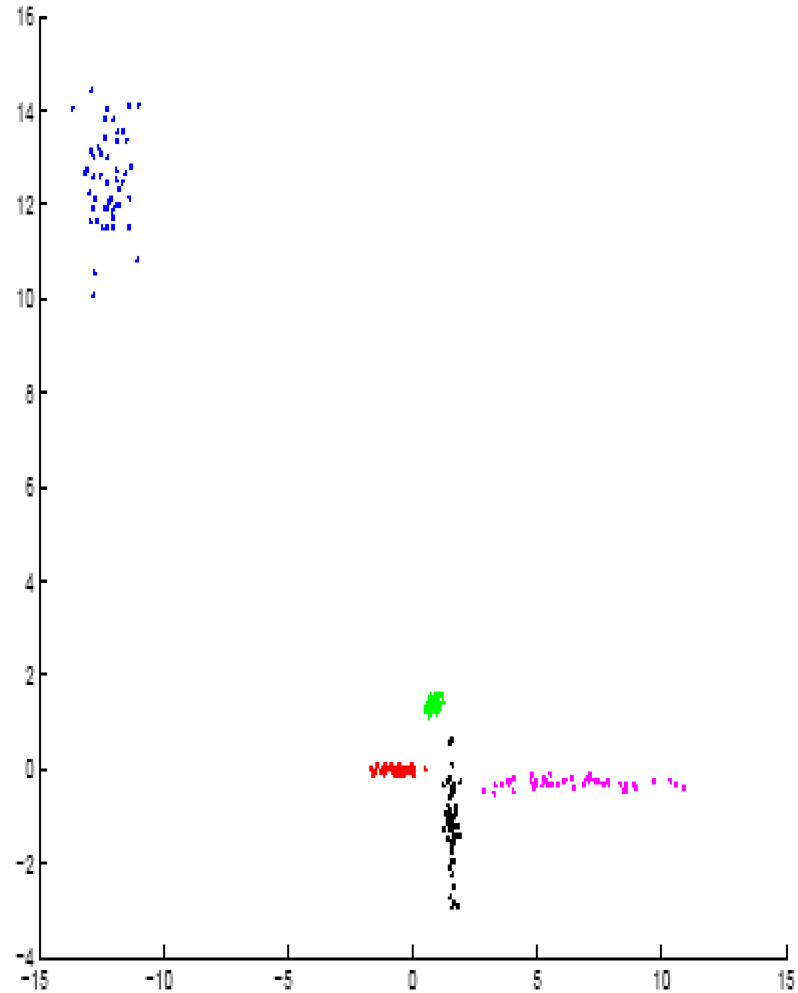
((r4 Lake () (Fields1) p1) (Fields1 (Lake) (Road *) p2) (Fields3 () (River Town Road *) p3) (Town () (swamp2 River Field1) p8) (River () (Fields3 Town Swamp2 Swamp1 Fields2 *) p7) (Swamp2 () (Town Road River *) p6) (Swamp1 () (River Fields *) p5) (Fields2 () (River Swamp1 *) p4))

Probability of image:

$P(Lake|r_4)P(p_1)P(Field|r_3)P(p_2)P(Field|r_2)P(p_3)P(Field|r_1)P(p_4)P(Swamp|r_7)P(p_5)P(Swamp|r_8)P(p_6)$
$P(River|r_6)P(p_7)P(Town|r_9)P(p_8)$

Segmenting the rule sets

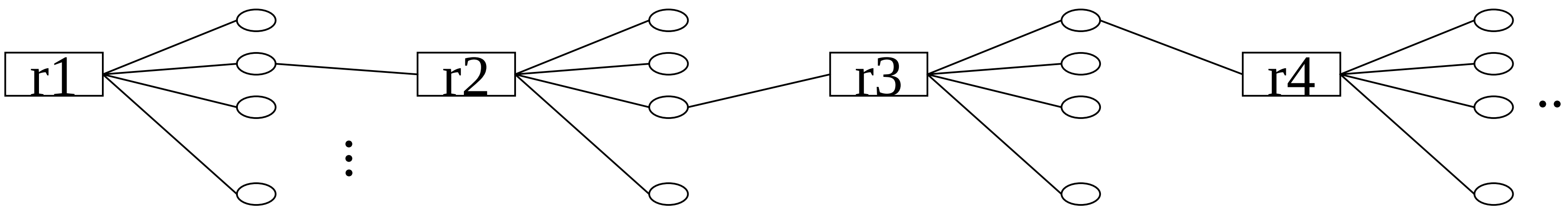




Network Search Parse

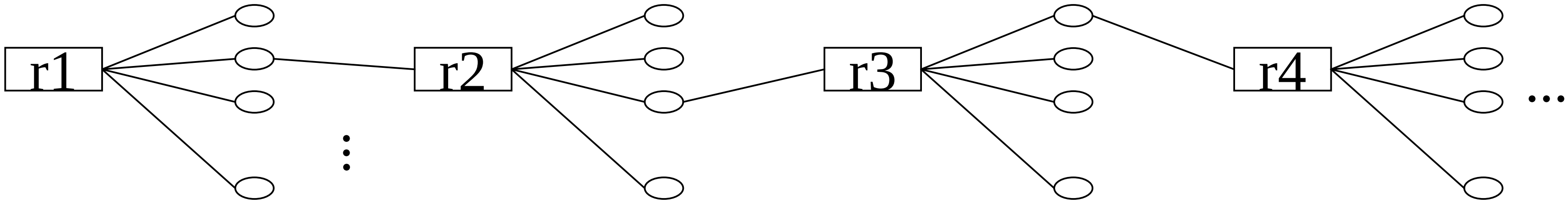
- Find parses in order or probability.
- Keep sorted list of partial parses (most probably first):
  - < bindings, unprocessed regions, probability>
- Start with:
  - (<(), (r1,r2,r3,r4,r5,r6,r7,r8,r9), 1.0>)
- At each step extend the most probable:
  - (<(r2=river, r5=swamp, r8=road, r6=field, r9=town) (r2,r3,r4,r5,r6,r7,r8,r9) 0.5> ...)
- When applying a rule bound regions must match, unbound regions are bound.
- First attempt to extend a parse that has a null "unprocessed regions" is the most probably parse.

Network Search Performance



- At each stage if there are m possible labelings of the region, and for each labeling if there are k rules, then for an image with n regions the cost of the network search parsing algorithm is:
  - $O((k*m)^n)$
- Even with only 9 regions, 9 rules, and 6 possible labelings per region there are of the order of 10^{15} candidates.
- Algorithm only terminates on VERY small examples.

Monte-Carlo Parse



- Select a complete parse at random as follows:

```
(dotimes (i N)
   (start-new-parse)
   (dolist (r region-list)
       (setq l (select-at-random (possible-labels-of r)))
       (setq r (select-at-random (rules-that-generate l))))
   (store-random-parse))
```

- Most frequently occurring parse will approach the most probable parse as N is increased.
- How big does N have to be?

Example Monte-Carlo Parse

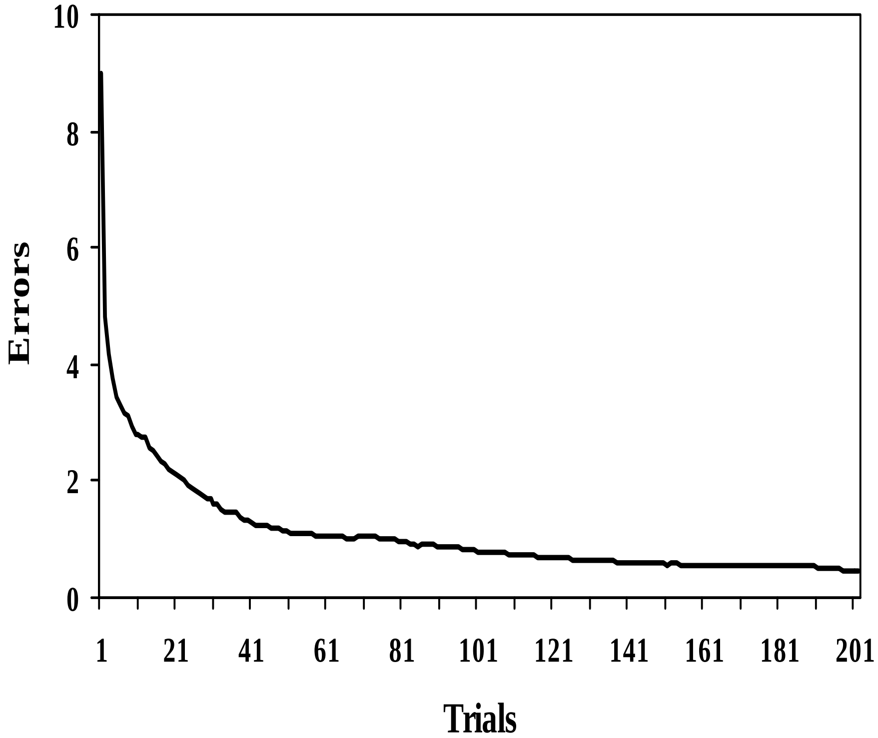
```
>> (parse-image-mc *all-regions* *rules* *region-optical-evidence*)
(((L1 . LAKE) (F1 . FIELD) (IM . IMAGE1) (RD . RIVER)
(S2 . SWAMP) (F3 . ROAD) (TN . TOWN) (F2 . RIVER) ...) NIL 4.2075E-9)

>> (dotimes (i 100) (next-parse-mc))
NIL
>> (first (setq *monte-carlo-parses* (sort *monte-carlo-parses* by-third)))
(((L1 . LAKE) (IM . IMAGE1) (S2 . SWAMP) (F1 . FIELD)
(RD . ROAD) (TN . TOWN) (F3 . FIELD) (RV . RIVER) ...) NIL 1.5147E-6)

>> (dotimes (i 100) (next-parse-mc))
NIL
>> (first (setq *monte-carlo-parses* (sort *monte-carlo-parses* by-third)))
(((F2 . FIELD) (S2 . SWAMP) (IM . IMAGE1) (F1 . FIELD)
(L1 . LAKE) (S1 . SWAMP) (RV . RIVER) (RD . ROAD) ...) NIL 2.4257475E-6)
>> (dotimes (i 100) (next-parse-mc))
NIL

>> (first (setq *monte-carlo-parses* (sort *monte-carlo-parses* by-third)))
(((F2 . FIELD) (S2 . SWAMP) (IM . IMAGE1) (F1 . FIELD)
(L1 . LAKE) (S1 . SWAMP) (RV . RIVER) (RD . ROAD) ...) NIL 2.4257475E-6)
>>
```

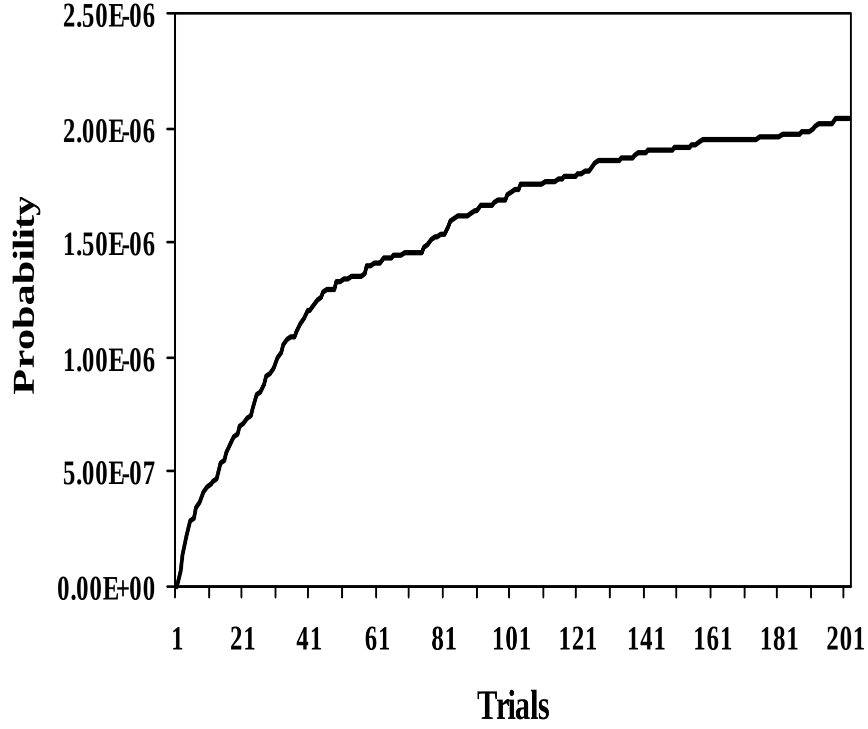
Monte-Carlo Performance

- Iterate until standard deviation < ε
  - As each sample is generated compute its probability.
  - Compute the standard deviation of the sample probabilities.
- We can make the error arbitrarily small by picking arbitrarily small ε.
- Best parse is the one from the sample with the highest probability.

```
(while (> (standard-deviation samples) epsilon)
  (start-new-parse)
  (dolist (r region-list)
     (setq l (select-at-random (possible-labels-of r)))
     (setq r (select-at-random (rules-that-generate l))))
  (store-random-parse))
```

Monte-Carlo Parsing Performance





Example of correctly parsed image