6.5.1 Mutually Unbiased Bases

We note that now we will consider our vectors to be complex valued, rather than real valued, but all of the results above hold for either case.

Consider the following $2d$ vectors: the d vectors from the identity basis and the d orthonormal vectors corresponding to columns of the Discrete Fourier Transform F. Since these basis are both orthonormal the vectors in question are unit-norm and within the basis are orthogonal, it is also easy to see that the inner product between any two vectors, one from each basis, has absolute value $\frac{1}{\sqrt{d}}$, meaning that the worst case coherence of this set of vectors is $\mu = \frac{1}{\sqrt{d}}$ this corresponding matrix $[I \ F]$ is RIP for $s \approx \sqrt{d}$.

It is easy to see that $\frac{1}{\sqrt{d}}$ coherence is the minimum possible between two orthonormal bases in $\mathbb{C}^d$, such bases are called unbiased (and are important in Quantum Mechanics, see for example [BBRV01]) This motivates the question of how many orthonormal basis can be made simultaneously (or mutually) unbiased in $\mathbb{C}^d$, such sets of bases are called mutually unbiased bases. Let $\mathcal{M}(d)$ denote the maximum number of such bases. It is known that $\mathcal{M}(d) \leq d + 1$ and that this upper bound is achievable when d is a prime power, however even determining the value of $\mathcal{M}(6)$ is open [BBRV01].

Open Problem 6.2 *How many mutually unbiased bases are there in 6 dimensions? Is it true that $\mathcal{M}(6) < 7$?*[29]

Reference

[BBRV01] S. Bandyopadhyay, P. O. Boykin, V. Roychowdhury, and F. Vatan. A new proof for the existence of mutually unbiased bases. *Available online at arXiv:quant-ph/0103162*, 2001.

[29]The author thanks Bernat Guillen Pegueroles for suggesting this problem.