

PROFESSOR: Uncertainty. When you talk about random variables, random variable Q , we've said that it has values Q_1 up to, say, Q_n , and probabilities P_1 up to P_n , we speak of a standard deviation, ΔQ , as the uncertainty, the standard deviation. And how is that standard deviation defined? Well you begin by making sure you know what is the expectation value of the-- or the average value of this random variable, which was defined, last time, I think I put braces, but bar is kind of nice sometimes too, at least for random variables, and it's the sum of the P_i times the Q_i .

The uncertainty is also some expectation value. And expectation value of deviation. So the uncertainty squared is the expectation value, sum over i , of deviations of the random variable from the mean. So you calculate the expected value of the difference of your random variable and the mean squared, and that is the square of the standard deviation.

Now this is the definition. And it's a very nice definition because it makes a few things clear. For example, the left hand side is ΔQ squared, which means it's a positive number. And the right hand side is also a positive number, because you have probabilities times differences of quantities squared. So this is all greater and equal to zero. And moreover, you can actually say the following.

If the uncertainty, or the standard deviation, is zero, the random variable is not that random. Because if this whole thing is 0, this ΔQ squared, ΔQ squared must be 0 and this must be 0. But each term here is positive. So each term must be 0, because of any one of them was not equal to zero, you would get a non-zero contribution. So any possible Q_i that must have a P_i different from 0 must be equal to \bar{Q} . So if ΔQ cubed is equal to 0, Q_i is equal to \bar{Q} as not random anymore.

OK, now we can simplify this expression.

Do the following. By simplifying, I mean expand the right-hand side. So sum over i , $P_i Q_i$ squared, minus 2 sum over i , $P_i Q_i \bar{Q}$ plus sum over i , $P_i \bar{Q}$ squared. This kind of thing shows up all the time, shows up in quantum mechanics as well, as we'll see in a second. And you need to be able to see what's happening. Here, you're having the expectation value of Q_i squared. That's the definition of a bar of some variable, you'd multiply with variable by the exponent of [INAUDIBLE].

What is this? This a little more funny. First, you should know that \bar{Q} is a number, so it can go out. So it's minus 2 \bar{Q} . And then all that is left is this, but that's another \bar{Q} . So it's another \bar{Q} . And here, you take this one out because it's a number, and the sum of the probabilities is 1, so it's \bar{Q} squared as well. And it always comes out that way, this minus 2 \bar{Q} squared plus \bar{Q} squared. So at the end, ΔQ , it's another famous property, is the mean of the square minus the square of the mean.

And from this, since this is greater or equal than 0, you always conclude that the mean of the square is always bigger than the-- maybe I shouldn't have the i here, I think it's a random variable Q squared. So the mean, the square of this is greater or equal than \bar{Q} squared.

OK. Well, what happens in quantum mechanics, let give you the definition and a couple of ways of writing it. So here comes the definition. It's inspired by this thing. So in quantum mechanics, permission operator Q will define the uncertainty of Q in the state, Ψ squared as the expectation value of Q squared minus the expectation value of Q squared.

Those are things that you know in quantum mechanics, how you're supposed to compute. Because you know what an expectation value is in any state Ψ . You so Ψ star, the operator, Ψ . And here you do this thing, so it's all clear. So it's a perfectly good definition. Maybe it doesn't give you too much insight yet, but let me say two things, and we'll leave them to complete for next time.

Which is claim one, one, that ΔQ squared Ψ can be written as the expectation value of Q minus absolute expectation value of Q squared. Like that. Look. It looks funny, and we'll elaborate this, but the first claim is that this is a possible re-writing. You can write this uncertainty as a single expectation value. This is the analog of this equation in quantum mechanics.

Claim two is another re-writing. ΔQ squared on Ψ can be re-written as this. That's an integral. Q minus \bar{Q} and Ψ . Look at that. You act on Ψ with the operator, Q , and multiplication by the expectation value of Q . This is an operator, this is a number multiplied by Ψ . You can add to this on the [? wave ?] function, you can square it, and then integrate. And that is also the uncertainty. We'll show these two things next time and show one more thing that the uncertainty vanishes if and only if the state is an ideal state of Q . So If the state that you are looking for is an ideal state of Q , you have no uncertainty. And if you have no uncertainty, the state must be an ideal state of Q . So those all things will come from this

planes, that we'll elaborate on next time.