PROFESSOR: Today we want to discuss wave packets. We have studied already the motivations that led to the Schrodinger equation and we even included the potential that described very non-trivial dynamics. With that Schrodinger equation, you could jump and solve the hydrogen atom, you could go and do the harmonic oscillator and all kinds of things. But we might as well develop, still, our tools and intuition of what's going on. And quite a lot of intuition comes from the free particle. And the free particle is interesting because you can go beyond the de Broglie wavelengths and the de Broglie waves. You had a single plane wave representing a particle with momentum and with energy, but now you become more sophisticated and consider the superposition of those waves. So we'll begin by discussing the wave packets and uncertainty.

So it's our first look into this Heisenberg uncertainty relationships. And to begin with, let's focus it as fixed time, t equals zero. So we'll work with packets at t equals zero. And I will write a particular wave function that you may have at t equals 0, and it's a superposition of plane waves. So it would be e to the ikx. You sum over many of them, so you're going to sum over k, but you're going to do it with a weight, and that's 5k. And there's a lot to learn about this, but the physics that is encoded here is that any wave at time equals 0, this psi of x at time equals 0, can be written as a superposition of states with momentum h bar k. You remember e to the ikx represents a particle or a wave that carries momentum h bar k.

So this whole idea here of a general wave function being written in this way carries physical meaning for us. It's a quantum mechanical meaning, the fact that this kind of wave has momentum. But this phi of k, however, suppose you know this wave function at time equals 0. Phi of k is then calculable. Phi of k can be determined, and that's the foundation of what's called Fourier's theorem, that gives you a formula for phi of k. And it's a very similar formula. 1 over 2 pi, this time an integral over x. So you take this of psi of x0 that you know and then multiply by e to the minus ikx. Integrate over x, and out comes this function of k. So if you know phi of x0, you know phi of k. You can calculate this interval and you can rewrite phi of x0 as a superposition of plane waves. So that's how you would do a Fourier representation.

So somebody can give you an initial wave function, and maybe it's a sine function or a Gaussian or something, then what you would do if you wanted to rewrite it in this way, is calculate phi of k, because you know this psi, you can calculate this integral, at least with a computer. And once you know phi of k, you have a way of writing psi as a superposition of

plane waves.

So we've talked about this before, because we were doing wave packets before and we got some intuition about how you form a wave packet and how it moves. Now we didn't put the time dependence here, but that can wait. What I wish to explain now is how by looking at these expressions, you can understand the uncertainties that you find on the wave function, position, and momentum uncertainties, how they are related. So that is our real goal, understanding the role of uncertainties here. If phi of k has some uncertainty, how is the uncertainty in psi determined? So that's what we're looking for. So relationship of uncertainties.

Now as before, we will take a phi of k, that we've usually be in writing, that depends on k and it's centered around some value k0. It's some sort of nice, centered function. And it has then, we say, some uncertainty in the value of the momentum. That is this signal, this phi of k that we're using to produce this packet. It has some uncertainty, it's not totally sharp, it's peaked around k0 but not fully sharp. So the uncertainty is called delta k and it's some typical width over here. Delta k is then uncertainty.

Now it's not the purpose of today's lecture to make a precise definition of what the uncertainty is. This will come later. At this moment, you just want to get the picture and the intuition of what's going on. And there is some uncertainty here, perhaps you would say, look at those points where the wave goes from peak value to half value and see what is the width. That's a typical uncertainty. So all what we're going to do in these arguments is get for you the intuition. Therefore, the factors of 2 are not trustable. If you're trying to make a precise statement, you must do precise definitions. And that will come later, probably in about one or two lectures. So at this moment, that's the uncertainty, delta k. And let's assume that this phi of k is real. And its peaked around k0 uncertainty delta k.

Now what happens with psi of x? Well, we had our statements about the stationary phase that you already are practicing with them for this homework. If you want to know where this function peaks, you must look where the phase, this phi-- we say it's real, so it doesn't contribute to the phase-- where the phase, which is here, is stationary, given the condition that it should happen at k0. The only contribution to the integral is basically around k0.

So in order to get something, you must have a stationary phase, and the phase must be stationary as a function of k, because you're integrating over k. And the phase is kx, the derivative with respect to k of the face is just x, and that must vanish, therefore, so you expect

this to be peaked around x equals zero. So the x situation, so psi of x0 peaks at x equals 0. And so you have a picture here. And if I have a picture, I would say, well it peaks around the x equals 0. So OK, it's like that. And here we're going to have some uncertainty. Here is psi of x and 0, and here is x.

And let me mention, I've already become fairly imprecise here. If you were doing this, you probably would run into trouble. I've sort of glossed over a small complication here. The complication is that this, when I talk about the peaking of psi, and you probably have seen it already, you have to worry whether psi is real or psi is complex. So what is this psi here? Should it be real? Well actually, it's not real. You've done, perhaps, in the homework already these integrals, and you see that psi is not real.

So when we say it peaks at x equals 0, how am I supposed to plot psi? Am I plotting the real part, the imaginary part, the absolute value? So it's reasonable to plot the absolute value and to say that psi absolute value peaks at x equals 0. And there will be some width again here, delta x, width. And that's the uncertainty in psi of x. So the whole point of our discussion for the next 10 minutes is to just try to determine the relation between delta k and delta x and understand it intuitively.