PROFESSOR: Now, what should happen? Somehow, this equation probably has solutions for all values of the energy, but those solutions diverge and are not normalizeable. It's the kind of thing you will find with a shooting method that you're doing with your computer. Count the solution and it suddenly diverges up or diverges down and cannot be normalized. But for some specific values, it can be normalized. So what we need is an intuition why this differential equation has normalizeable solutions, only if the energies, curly E, take specific values. That's intuition that we need.

For that, we'll look at the equation a little closer, and try to understand what happens at the place where it can get in trouble, which is large X. That's where you expect it to get in trouble. So what does this equation become as u goes to plus minus infinity? Well this equation, at that stage, becomes like this. The second phi, the u squared is roughly equal to u squared phi. Because e is a constant, it doesn't blow up, so the differential equation, the terms that are supposed to be largest in this right hand side, is the u squared.

So how does this look as a solution? And you can see that that's definitely not like a power solution. If you have a power of u, say phi equals u to the n, after u differentiates, the power goes down. But here it goes up. You have a u to the end. The relative is supposed to give you u to the n plus 2, but that doesn't work. So that's not a power solution. So it has to be different.

So what it is, you can try a phi. And what it should be is of the form e to the u squared. It's kind of like that, because this is the function that when you're differentiate, you bring down the derivative of this quantity, which is a u. When you differentiate again, you can bring another u to get the u squared, or you can differentiate this one. But if you differentiate the thing that is in the bottom, you get something that diverges less. So morally speaking, this function is about right. So I'll put here, for example, e to the alpha over 2. And I'll even put here a uk.

Can this work? A u to the k times alpha u squared. Well if we take two derivatives, if we take one derivative, if I differentiate the u to the k, I get u to the k minus 1. I lose powers of u. If I differentiate the exponential, I can get alpha times u times the same function. See, that's, roughly speaking, what's happening. You differentiate the thing that diverges the most.

So if you differentiate twice, each time you differentiate you get a factor of alpha times u squared phi, roughly. This is plus subleading. When you differentiate a function and you're wanting to show the most divergent thing, then you-- because we're looking at the most divergent part, you will always differentiate this, and this u to the k is really a spectator, it doesn't do anything, because when you differentiate that, you get something much smaller, that doesn't matter.

So yes, with these exponentials, we get something like this. Beside double pronged should have been here. And therefore, you see that alpha is plus minus 1. Alpha is plus minus 1, and those are likely to be approximate solutions as x goes to infinity. So we could expect solutions for alpha equals 1, and I will write that it, and all this, I should say is always as u goes to infinity. So always as u go to infinity, all of this in this blackboard.

So also as u go to infinity, we would expect maybe solutions of the form A u to the k, e to the minus u squared over 2 plus B u to the k e to the u squared over 2. The two values of alpha equal plus minus 1 are the possibilities for these two equations to match. So you would expect things like this to be solutions.

And here you are seeing the beginning of the danger. Well a minus u squared over 2 times a power sounds pretty good, but a u squared over 2 times a power sounds pretty bad. So maybe this is what we want to happen. This is not an exact solution of anything yet. We're just looking at u going to plus and minus infinity, and maybe we'll have such a behavior or such behavior. But we want this one, otherwise we will never be able to normalize it.

So here it is. Without any loss of generality and inspired by this, this analysis is absolutely crucial, you see, we're following a very logical procedure. Cleaning the equation then looking where the divergence would happen and learning something about the form of the solution. Now without any loss of generality, I can write, I will write 5x is going to be a-- not of x anymore. U, h of u times e to the minus u squared over 2.

So it's an an sat, but it's without any loss of generality, because you can always write any function as another function times e to the minus u over 2, because you take the function, you multiply by e to the plus u over 2 and e to the minus u over 2 and it's written like that. But this should be nice, because it has sort of the right behavior already. And here is the hope. The hope is that this function now is a proxy for phi. If you know h, you know phi, which is what you want. And this function, hopefully, won't diverge. This will be into [INAUDIBLE].

So if this function doesn't diverge, it will be a great thing. In particular, we could hope that h of u is a polynomial. You see, if somebody with have come and said, look at that equation. Could that be a polynomial? A polynomial is something that ends up to some power x to the 20 or x to the 30, but it doesn't go up forever. An exponential has all powers. This equation doesn't have a polynomial solution. No polynomial will ever solve this. But now that you've isolated the divergence, there is a hope that a polynomial will work.

So for doing that, exploring that hope, I now have to substitute-- this is no assumption-- the differential equation for phi implies a differential equation for h, you just substitute this and look at it. That's a one line computation or two line computation. I'll give the answer. So what is the differential equation for h? So back in star, you'll get the second h, the u squared minus 2u dh du plus e minus 1h equals zero.

So you substitute that into this equation and you get this differential equation. And now this is a differential equation for h. We hope it has a polynomial solution. You will see that it wants to have a polynomial solution, but it doesn't quite make it. And then you will discover quantization helps, and suddenly you get the other normal solution and everything works out.