PROFESSOR: We start with the stationary states. In fact, stationary states are going to keep us quite busy for probably a couple of weeks. Because it's a place where you get the intuition about solving Schrodinger's equation. So the stationary states are simple and useful solutions of the Schrodinger equation, very nice and simple.

So what are they by definition? Well, we begin with a definition. And the intuition of a stationary state will follow. See the word stationary is not the same as static. Stationary is something that maybe it's kind of moving, but things don't change. Static is something that's just not moving. Stationary states have time dependence. It is very simple, as we will see. So, your definition of a stationary state has a factorized space and time dependencies.

So this psi of x and t is a stationary state. If you can write it as a product of a function of time times a function of position. And now, I will try to be careful about this. Wave functions that have position and time will have this bar at the bottom. Wave functions that don't have x will not have it. If I slip on that, please let me know.

So this is a stationary state, but a stationary state has factorized space and time dependencies and solves the Schrodinger equation-- the solution of Schrodinger's equation. So what we need to understand is what this factorized dependence tell us for the Schrodinger equation. So this stationary state has time dependence.

But the thing that makes them stationary is that if you look at some observable, a Hermitian operator, and you say, OK, the state has time dependence, so maybe my observable will have time dependence. No. The observables don't have time dependence. That is the nice thing about stationary states.

So, what we call time independent observables which are all observables that are familiar [INAUDIBLE]-- Hamiltonian, the momentum, the precision, the angular momentum. Time independent observables have no time dependence. And it kind of looks simple when you write it like that. Time independent means don't have time dependence.

But you've seen that d dt of the expectation value of x is equal to p over m, or the sum of p over m, the velocity. And here it is-- a time independent observable that does have time dependence. So the observable is time independent, but expectation value have no time dependence in their expectation values.

The time dependence comes from the state-- the state, the psi of x and t have time dependence, and sometimes it just doesn't drop out. But for stationary states, it will drop, as you will see. So, time independent observables have no time dependence in their expectation values.

OK. So enough of saying things. And let's just get to them. So we look at the Schrodinger equation, i h-bar d dt of psi of x and t is equal to h-bar psi of x and t. And just to remind, this minus h squared over 2m d second d x squared plus V of x. And I will consider states that have just that at this moment.

Otherwise, it's not so easy to get time-dependent-- to get stationary states. If you have a potential that has time dependence, we kind of do the nice thing that we're going to do. So we're going to look only at time independent potentials. So, V of x, like this, times psi of x and t. OK. So what we do next is to simply substitute the value of the wave function into the differential equation and see what we get.

So on the left hand side, we will get i h-bar The psi of x goes out but you have d dt, Now a normal derivative of g of t. And now, this factor, H of psi acts on these two things. Imagine the function of time times the function of x sitting here.

Well, the function of time can be moved out. So the function of time can be moved across the Hamiltonian operator. It doesn't do anything to it. So we'll have g of t times H-hat of the psi of x. This is H-hat. And because we had no time dependence in the potential, our assumption, this whole thing is a function of x.

All right. Next step. Divide this whole equation by the total wave function. Divide by psi. Well, if you divide by psi, you cancel the middle psi here, and you get the 1 over g. So you get i h-bar 1 over g dg dt is equal-- on the right side, you cancel the g and you get a 1 over psi of x H-hat psi of x.

And now you look at this equation. And this equation is saying something very strange. The left hand side is a function of time only. The right hand side is a function of space only. How can a function of time be equal to a function of space? The only way this can be is if both are not a function of what they were supposed to be. They're just numbers. Any function of time cannot be equal to a function of space, in generality. It just doesn't make sense.

So each side must be equal to a constant, and it's the same constant. So each side, this is all

equal to a constant. And we'll call the constant E. And this E has units of energy. E equal to be a real constant with units of energy.

You can see the units because the Hamiltonian has units of energy. And whatever psi units it has-- whatever every unit psi has, they cancel. Here, whatever units g has, they cancel. And h-bar over time is units of energy, like in energies equal h-bar omega.

So it has units of energy. The only thing that you may be could say, why real. Quantum mechanics loves complex numbers. So why don't we put the complex E? We'll see what trouble we get if you choose something that is complex.

So here we go. It's a real quantity to be-- let's do it real for the time being. And let's solve the first equation. The left hand side, i h-bar dg dt is now equal to gE, or E, where E is a number and g is a function of time from where g of p is equal to constant E to the minus iEt over h-bar. Let's just check it works correctly.

It's a first order differential equation. Just one function of integration. If you guess the answer, must be the answer. And that's the time dependence of a stationary state. It's exponential minus iEt over h-bar.

What about the other equation? The other equation has become H psi of x equals E psi of x. Or, we should write at least once, minus h squared over 2m-- did I make a mistake? No, I didn't-- d second dx squared-- I got this normal derivatives here because this is just a function of x-- plus V of x psi is equal to E psi of x.

This is the same equation that I'm boxing twice, because it's written in those two ways. And both ways are very important. And this is part of solving for stationary state. You've solved for g of t. The time dependence was easy to solve for, but the x dependence is complicated, in general. There, you have to do some work. You have to solve a differential equation. It's not that easy.

So many people-- most people-- call this the time independent Schrodinger equation. So that's the time independent Schrodinger equation, where H psi equal E psi. And as you can imagine, solving this differential equation can be challenging, or sometimes very interesting because it may be that, as far as the first equation is concerned, of what we did here, we don't know what this number E is. But it may be that the only reasonable solutions that this equation has are for some values of E.

The analogy with matrices should tell you that's probably what's going to happen. Because you remember eigenstates and eigenvalues of matrices are peculiar numbers. If you have a matrix, they're peculiar eigenvalues. So this equation is an eigenfunction equation. And it's possible that it has the solution for some particular values of the energy.

Let me write the whole solution then. If you've solved these two things, the whole solution psi of x and t is now a constant times psi of x times e to the minus iEt over h-bar, where this psi of x solves this equation. So this is the stationary state.

How about normalizing the stationary state? Can we do that? Well, if we try to normalize it-- psi star of x and t and psi of x and t dx, and you set this equal to 1. This should be the case, because this should be interesting solutions of the Schrodinger equation. We expect that we could do particles with them. And we can start wave packets or peculiar states with them.

And let's see what we get here. Maybe I should know. I'm really [INAUDIBLE]. I'm going to erase that constant C here. Since we want to normalize this, we will think of this as a normalization of psi. When we try to normalize psi, we'll be normalizing middle psi, as you will see here. There's no need to put that constant there.

So what do we get here? We get integral dx psi star of x and t, so you have psi of x star. And now you could say it's E to the iEt over h-bar. That's the complex conjugate.

Now, on the other hand, suppose-- I'll do this this way. [INAUDIBLE] of this other term is psi of x into the minus iEt over h-bar. And now the good thing about this, you see this integral should be normalized to 1 to make sense. And it's a great thing that the time dependence drops out.

And it would not have dropped out if the energy had not been real. If the energy was not real, I would have had to put here E start. And here I would have had E star minus E and some function of time. And how can a function of time be equal to 1? Would be a problem. We would not be able to normalize this wave function.

So E must be real because otherwise we don't cancel this time dependence, which happily, when it cancels, it just tells you that the integral dx of psi star of x psi of x must be 1, which is a very nice thing. So in a stationary state, the normalization condition for a full time dependent stationary state is that the spacial part is normalized.