PROFESSOR: That's how it looks, a resonance. You can see it basically in the phase shift. And great increase of the phase of almost minutely pi over a very small change of energy. And it should [INAUDIBLE] with a very big [INAUDIBLE].

So this is how it looks. And I want to now proceed, after if there are some questions, of how do we search for residences a little more mathematically rather than plotting them. How could I write an equation for a resonance. Cannot say, oh, the phase changes fast. Well, that's not a very nice way of saying it. It's good. It's intuitive. But we should be able to do better.

So how do I find resonances? So let's model resonances a little bit. How do we find resonances? So let's model this behavior. By that is writing a formula that is simple enough that seems to capture what's happening. And that formula's going to inspire us to think of resonances perhaps a little more clearly.

So suppose you have a resonance near k equal alpha. I claim the following formula would be a good way to represent the resonance? We would say that tan delta is equal to beta over alpha minus k. Or-- yeah, we would say that. [INAUDIBLE] Or if you wish, delta is tan minus 1 of beta over alpha minus k.

Why is that reasonable? It's a little surprising, but not that surprising. You see that-- delta is equal to minus pi over 2. The tangent of delta goes to infinity. So there's something going on here in which you have this property. So let's plot this. So let's plot beta over alpha minus k. You need a clock to understand this.

So this is k, and we're plotting this quantity. Well, it's going to go crazy at k equal alpha. That we know. When k is less than alpha, I'm going to assume that alpha and beta are positive. They both have units of k. And when k is less than alpha-- we begin here-- then this denominator is positive, the numerator is positive, ratio is positive. It's small, maybe.

And then suddenly, when k reaches alpha it goes to infinity. So it's going to be like that. Now, it actually is true that when k differs by alpha by beta, it reaches value 1. So here is alpha minus beta. That point it reaches value 1.

So if I want this thing to be very sharp, I need beta to be small so that it's little until it reaches beta within-- distance beta within alpha, and then it shoots up. So I want beta to be small for sharp behavior. On the other hand here, it goes the other way. It goes from minus infinity back to 0, and has value minus 1 at alpha plus beta.

So within minus beta, and beta off of the center alpha, most of the things happen. If we plot now the tangent of this, or the arctangent of this, tan minus 1 of beta, alpha minus k, well, if the tangent of an angle is very little, the angle can be taken to be very little. At this point, it will reach pi over 2, so the angle is little, will go to pi over 2, and then quickly becomes larger than pi over 2, you're thinking tangents.

So the tangent is going up, is blowing up at pi over 2. Then continuously, it goes to minus pi over 2, and then continuously goes to 0 so it reaches pi. So this is the behavior of delta. Delta is this tan minus 1 of beta over that. And delta is doing the right thing. It's doing this kind of behavior.

There is a shift. I could add a constant here to produce this shift, but it's not important at this moment. The resonance is doing this thing, up to a total shift of pi that doesn't change the tangent of an angle. So this is one way of modeling what's happening to the phase shift near our resonance. So let's explore it a little more.

I can do a couple of calculations. For example, I can compute what is d delta dk at k equals alpha. That's should be a nice quantity. Is a derivative of the face. At the resonance, at the position alpha of the resonance. So here's delta, here is k, and there's the derivative of this k.

And how should it be? Well, basically, the phase changes by amount pi over a distance beta or 2 beta. So this must be a number divided by beta. You can calculate this derivative from this equation. It's a nice exercise. It's actually just 1 over beta. That's a result. 1 over beta.

The other quantity that is nice to understand is how does this scattering amplitude behave near the resonance. So what is the value of as squared? Oh, that's the absolute value of psi s squared, which is sine squared delta. That's the same thing as As squared. Well, you know what is the tangent of delta? A little trigonometric play should be able to do it, and can give you the sin squared delta.

And here is the answer. It's beta squared over beta squared plus alpha minus k squared. Kind of a nice, almost bell shape. Of course, it's polynomial, but it looks a little like just a nice symmetric shape around alpha equal k.

Now, this division is so famous it has been given a name. It's called the Breit-Wigner distribution. Breit-Wigner. But it's described as the Breit-Wigner distribution, and it's usually referring to terms of energy. Of energy, not momentum. So-- and it's-- what should happen to scattering amplitude in general when you have a resonance.

So the way to do this calculation now is to say, well, what is alpha minus k? Let's try to relate it to the energy minus the energy at k equal alpha. Well, this is h squared k squared over 2 m minus h squared alpha squared over 2 m, which is h squared over 2 m, k squared minus alpha squared. On the other hand, I have here alpha minus k squared. I don't have k squared minus alpha squared.

So-- approximations. if the resonance is narrow enough, if beta is small, let's do an approximation. We do h squared over-- everybody knows this approximation, shouldn't be afraid of doing it. It's alpha-- how could I write it-- k minus alpha times k plus alpha. And the approximation is that all the interesting thing comes from the difference between k and alpha, how close k is to alpha.

So when k is close to alpha, all the dependence is going to be here. This is going to be about 2 alpha when k is near alpha. And if it's a little more than that, it doesn't matter, because it's [INAUDIBLE]. So this could be approximated to 2 alpha, and therefore this becomes h squared alpha over m times k minus alpha. So that's a little help.

Then size of s squared, doing a little more of algebra with the constants there. Probably you want to do it with two lines. It's tricky. It's really simple. It's always written in this form-- 1 over 4 gamma squared over e minus e alpha squared plus 1 over four gamma squared, and that's the so-called Breit-Wigner distribution.

And gamma is a funny constant here. We'll try to understand it better. 2 alpha beta h squared over m. It has to be something that depends on alpha and beta, because after all, we weren't modeling the resonance with alpha beta. So this curve is very famous. That's the distribution of the scattering amplitude over energies whenever you have a resonance. So we should plot it.

You have an e alpha. You have an e alpha plus gamma over 2 and e alpha minus gamma over 2. But the energy minus e alpha is equal to the gamma over 2, you get the gamma squared over 4 so the total amplitude goes down to 1/2 of the usual amplitudes. When the energy is equal to e alpha, you get 1. 1 for psi s squared.

But when the energy differs from e alpha by gamma over 2, you get half. So-- actually, I'm not sure of the deflection point, where it is. Probably not there. Or is it there? I don't know. I drew it as if it is there. So that's the distribution, and the width over here is gamma. So gamma is called the width at half power, or at half intensity. Yeah. Width-- the half width of the distribution.